Spectrum and reflection of spin waves in magnonic crystals with different interface profiles

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Using the transfer-matrix method, we have developed a theory of exchange spin waves in a thin cylindrical magnonic crystal (periodically layered all-ferromagnetic nanowire) with diffuse interfaces. The magnonic spectrum and the frequency dependence of the reflection coefficient of spin waves from a junction between a homogeneous magnetic nanowire and a magnonic crystal have been calculated and compared. Diffuse interfaces with linear and sinusoidal profiles of variation in the uniaxial anisotropy value have been considered, also allowing for asymmetry in the relative thicknesses of either main layers, or interfaces, or both. We have found that, although the thickness and profile of interfaces have a significant effect on the size and position of the magnonic band gaps, the smoothing of interfaces does not lead to disappearance of the band gaps. At the same time, the profiles and relative thicknesses of interfaces might provide additional means by which to design magnonic crystals with a desired magnonic spectrum.

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I. INTRODUCTION

Magnonic crystals are materials with periodically modulated magnetic parameters and represent spin-wave (magnon) counterparts of photonic,[1] phononic,[2] and plasmonic[3] crystals. The study of spin waves and magnonic crystals (so-called "magnonics") has been intensively growing recently as a new research field at the interface between nanomagnetism and electromagnetic metamaterials. So, many exciting results have recently been obtained in magnonics by experimental,[4–9] theoretical,[10–17] and computational[18–20] means.

Generally, the theory of waves in layered media is well developed. In particular, a number of exactly solvable models have been proposed and thoroughly investigated (see, e.g., Refs. 21 and 22 for review). In part, the advance in this and some other research directions has been aided by the remarkable analogies with the quantum-mechanical theory of an electron in a one-dimensional (1D) potential. The analogy can be drawn in some cases of 1D problems of spin-wave propagation in nonuniform magnetic fields[23–25] and also in the discussion of nucleation fields in composite magnets.[26–30] In the latter case, due to the dominating role of the anisotropy field, the analogy holds also for two-dimensional (2D) and three-dimensional (3D) problems. Nonetheless, in a general case, the results obtained for other excitations and quasiparticles (e.g., phonons or electrons) are not readily transferred to the case of magnonic crystals. This is mainly due to the complicating role of the long rage magnetodipole interaction, combined with the increased number of material parameters that affect the dispersion of magnons in such media. The situation is even more complicated in the case of 2D and 3D magnonic crystals, for which only few theoretical results have been published so far,[10–13] with strong approximations often made. Alternatively, the spectra of spin waves in magnonic crystals can be obtained from micromagnetic simulations (with only 1D case having been considered to date[18,19]), or using the dynamical-matrix method,[20] which can be thought of as micromagnetics in the reciprocal space. On the experimental side, there is a record of studies of periodic magnetic multilayers[31–33] while more recent investigations concentrated on planar arrays of magnetic elements[34–37] and magnonic crystals.[4–9]

The studies confirmed that the spectrum of exchange, magnetostatic, and dipole-exchange spin waves in magnonic crystals forms band structure, as opposed to the discrete spectrum observed in systems of noninteracting magnetic thin films or elements. The next issue to consider is how the magnonic band structure is affected by various kinds of imperfections, which are inevitably present in realistic samples and devices of magnonics, and how to suppress any destructive effects of such imperfections. So, in Refs. 38 and 39, Ignatchenko *et al.* studied the modification in the magnonic band spectrum due to random variations in the period of a magnonic crystal. Localization of spin waves by isolated defects was investigated in Refs. 40 and 41. Here, we again note the useful analogy between the problem of magnonic spectrum and the problem of nucleation.[29,42] In Refs. 39 and 43, the effects of the presence of damping and of its modulation in magnonic crystals were studied.

In addition, realistic multilayered and planar magnonic crystals are likely to have diffuse interfaces.[44] In other words, transitions between constituent layers of a magnonic crystal or between elements made of different materials[9] are likely to have finite thickness. Magnetic properties of such transition regions are generally different from those of the two adjacent materials and can continuously vary across the thickness of the interface. Alternatively, such interfaces could be created artificially, e.g., aiming to create a particular magnonic band spectrum. In Refs. 44 and 45, Ignatchenko *et al.* investigated propagation of waves in periodic multilayer materials with diffuse interfaces. The magnonic crystal was considered as a model magnonic crystal with a periodically modulated value of the uniaxial anisotropy constant. Both the spectrum and the scattering parameters (i.e., transmission and reflection coefficients) of spin waves were found to depend strongly upon the thickness of interfaces. In the model, the spatial modulation was described by a Jacobian elliptical sine function. However, the model is not solvable exactly, and so the perturbation theory had to be used instead, assuming a small modulation of the anisotropy value. In contrast, exactly solvable models were used in Refs. 46–48, to study the effect of diffuse interfaces upon the spectrum of spin waves propagating in magnonic crystals with an arbitrary large amplitude of modulation of the anisotropy value. The difference between results obtained using the two approaches can be drastic. For example, the first-order perturbation theory predicts vanishing of all $n>1$ band gaps for a sinusoidal magnonic crystal,[48] which is not the case in the exact solution in terms of Mathieu functions.[46,49]

Here, we address the question of the effect of the particular functional form of the variation of interface's properties across its thickness upon the magnonic band structure of a thin cylindrical magnonic crystal (periodically layered magnetic nanowire with a circular cross section). In particular, we compare models with linear and sinusoidal variations of the anisotropy value in the transition regions. The aim is to study the effect of thicknesses of the constituent layers and interfaces as well as the structure of the interfaces, upon the spectrum and reflection coefficient of spin waves from a semi-infinite magnonic crystal. The here presented study of the reflection of spin waves from a semi-infinite magnonic crystal will be particularly important for measurements and practical implementation of propagating spin waves in magnonic devices.

In Refs. 50 and 51, imperfect interfaces were modeled as regions with increased damping. It was found that the overall damping of spin-wave modes in magnonic crystals described by such models can vary from band to band much stronger than in the case of damping uniformly distributed within layers, as considered in Ref. 43. However, the damping of spin waves is beyond the scope of this particular study, where we focus on the functional profiles of interfaces.

II. MODEL

Due to space constraints, future networks of magnonic waveguides are likely to consist of waveguides with a narrow cross section.[15] Hence, we model the magnonic crystal as an infinitely long *wire* consisting of periodically alternated adjacent uniform layers of two types. The latter layer types differ only by the strength of the uniaxial anisotropy while the easy axis is parallel to the axis of the wire with the associated unit vector denoted as **n**. This assumption is a reasonable approximation, backed up by the recent progress in fabrication of multilayered magnetic nanowires.[52] So, in Ref. 52, the authors successfully grew multilayered Co nanowires in which only the strength of the uniaxial anisotropy constant was modulated. The exchange parameter α, the gyromagnetic ratio g, and the spontaneous magnetization M_0 are assumed to be constant throughout the sample.

Furthermore, we assume that the so called "main" layers of the magnonic crystal are separated by "transition" layers in which the strength of the anisotropy varies as

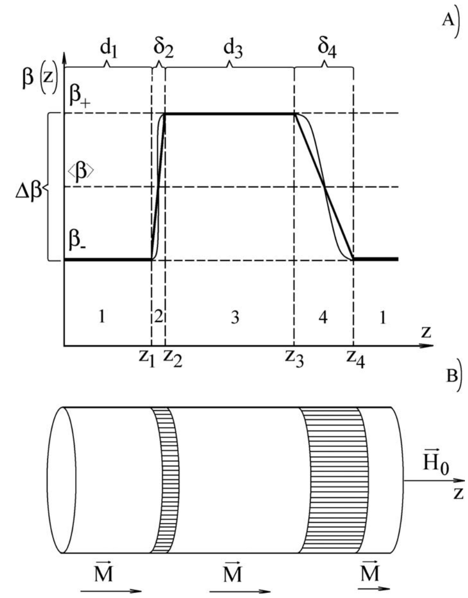

FIG. 1. (a) The coordinate dependence of the anisotropy value in the magnonic crystal is schematically shown for linear (thick solid line) and sinusoidal (thin solid line) profiles of the interfaces. (b) The geometry of the problem is schematically shown.

$$\beta(z)=\begin{cases}\beta_{1,3}=\langle\beta\rangle\mp\dfrac{\Delta\beta}{2} & z_{0,2}+lL<z<z_{1,3}+lL\\[2ex] \beta_{2,4}=\langle\beta\rangle\pm\dfrac{\Delta\beta}{2}\xi(z) & z_{1,3}+lL<z<z_{2,4}+lL,\end{cases}\tag{1}$$

where $\langle\beta\rangle=\frac{\beta_-+\beta_+}{2}$ and $\Delta\beta=|\beta_+-\beta_-|$ are the average value and the amplitude of modulation of the anisotropy, respectively, l is the number of the period, and $z_{1,2,3,4}$ are the coordinates of the layer boundaries within the period, as shown in Fig. 1. The upper and lower signs in the expression correspond to the first and second indices, respectively. L is the period of the magnonic crystal, and $d_{1,3}$ and $\delta_{2,4}$ are thicknesses of the main and transition layers, respectively. One can see that the model allows one to study not only asymmetry in the thicknesses of the main layers but also asymmetry in the profiles of the interfaces, e.g., magnonic crystals in which sharp and smooth interfaces alternate ($\delta_{2(4)}=0$, $\delta_{4(2)}\neq 0$). The Z axis is chosen to be parallel to **n**.

Equation (1) can describe both interface profiles studied here. For linear profile, one has

$$\xi(z)=\frac{z-z_{2,4}}{\delta_{2,4}}.\tag{2}$$

For sinusoidal profile, one has

$$\xi(z)=\cos\left[\pi\frac{z-z_{2,4}}{\delta_{2,4}}\right].\tag{3}$$

III. SPECTRUM

To describe dynamics of magnetization $\mathbf{M}(\mathbf{r},t)$ in the magnonic crystal, we use Landau-Lifshits equation[53]

$$\frac{\partial \mathbf{M}}{\partial t} = -g\left[\mathbf{M} \times \left\{[H+\beta(\mathbf{Mn})]\mathbf{n} + \mathbf{h}_m + \frac{\partial}{\partial \mathbf{r}}\left(\alpha\frac{\partial \mathbf{M}}{\partial \mathbf{r}}\right)\right\}\right], \tag{4}$$

where H is the bias magnetic field parallel to $\mathbf{n}$, $\mathbf{h}_m$ is the demagnetizing field, g is the gyromagnetic ratio, and α is the exchange constant. Assuming sufficiently strong bias and anisotropy fields, the magnetization in the ground state is uniform and parallel to the axis of the wire. To consider small amplitude spin waves, we can represent magnetization as

$$\mathbf{M}_j(\mathbf{r},t) = \mathbf{n}M_0 + \mathbf{m}_j(\mathbf{r},t), \tag{5}$$

where $\mathbf{m}_j$ are small deviations of magnetization ($|\mathbf{m}_j| \ll M_0$) in layer $j=1,\ldots,4$ from the ground state, assuming that the length of the magnetization vector is constant $|\mathbf{M}_j(\mathbf{r},t)| = M_0$. We assume that the diameter of the wire is smaller than the exchange length, and so, magnetization dynamics are uniform in the circular cross section of the wire.[54,55] Then, it is appropriate to describe the nonvanishing components of the dynamic demagnetizing field, which are perpendicular to the axis of the wire, in terms of ballistic demagnetizing factors. The static demagnetizing field in an infinite cylinder magnetized along its axis is already zero. Hence, one has $\mathbf{h}_m = -\overleftrightarrow{\mathbf{D}}\mathbf{M}$, where $\overleftrightarrow{\mathbf{D}}$ is the tensor of ballistic demagnetizing coefficients that has only two nonvanishing components $D_{xx} = D_{yy} = 2\pi$.

We linearize Eq. (4) using Eq. (5), introduce Fourier components $\mathbf{m}_j(z,t) = \mathbf{m}_{\omega,j}(z)\exp\{i\omega t\}$ where t and ω are time and frequency, respectively, and do a standard substitution of variables $\mu = m_x + im_y$. Then, we obtain the following equation that describes propagation of spin waves in each layer of the magnonic crystal

$$\frac{d^2\mu_j(z)}{dz^2} + k_j^2(z)\mu_j(z) = 0, \quad k_j(z) = \sqrt{\frac{\Omega - h - 2\pi - \beta_j(z)}{\alpha}}, \tag{6}$$

where $\Omega = \omega/gM_0$ and $h = H/M_0$.

Solutions of Eq. (6) and their derivatives must be continuous at the boundaries of the main and transition layers as well as everywhere else. This ensures that, first, in the regime of dominating exchange interaction, the magnetizations on the opposite sides of the boundary are parallel, and second, the normal component of the vector of the density of energy flux is continuous.[53] Besides, the solution of Eq. (6) must satisfy the condition of periodicity, i.e., solutions μ_j at the period boundaries $z=0$ u $z=z_4=L$ must differ only by a phase factor[56]

$$\mu(0) = e^{iKL}\mu(L), \tag{7}$$

where K is the quasiwave number (Bloch wave number).

To find the spectrum of spin waves, we use the method of transfer matrices $\hat{\mathbf{M}}_j$,[56] which link values of the dynamic magnetization in the beginning and the end of each layer

$$\left.\begin{pmatrix}\mu(z)\\ d\mu(z)/dz\end{pmatrix}\right|_{z_{j-1}} = \hat{M}_j \left.\begin{pmatrix}\mu(z)\\ d\mu(z)/dz\end{pmatrix}\right|_{z_j}. \tag{8}$$

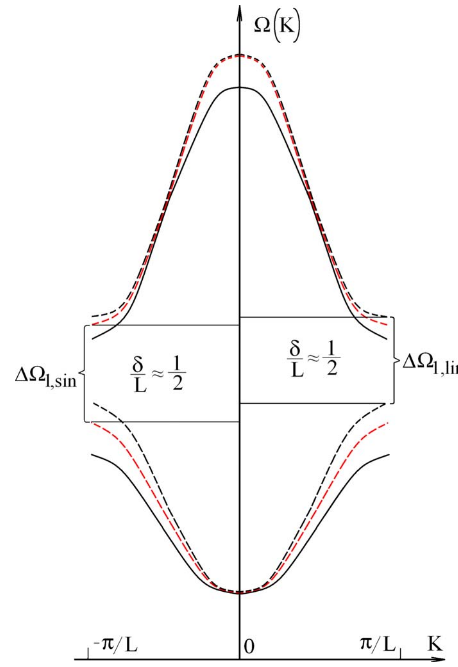

FIG. 2. (Color online) The two lowest bands of the spectrum of spin waves described by Eq. (9) are shown for magnonic crystals with the linear (black dotted line) and sinusoidal (red dotted line) interface profiles and for ideal magnonic crystals with sharp interfaces (solid black line), all with $\beta_+/\beta_- = 2$ and $\delta_2 = \delta_4 = \delta$. Here and in the following we assumed the value of the exchange constant of 3×10^{-12} cm^2, which is characteristic for Co.

Using Floquet-Bloch theorem Eq. (7), it is then easy to write the spectrum of spin waves in the magnonic crystal (Fig. 2) in the following form:

$$\cos(KL) = -2\tilde{M}, \tag{9}$$

where $\tilde{M} = \mathrm{Sp}(\hat{\mathbf{M}})$ [or $\tilde{M} = \mathrm{Tr}(\hat{\mathbf{M}})$] is given in the Appendix.

IV. REFLECTION COEFFICIENT

From the point of view of application of magnonic crystals in magnonic devices, e.g., as spin wave filters, it is important to know how spin waves are reflected from the boundary between a semi-infinite magnonic crystal and the adjacent semi-infinite uniform ferromagnetic medium. To calculate the coefficient of reflection of spin waves incident upon the boundary, we use the recurrent method proposed in Ref. 57 for investigation of scattering of neutrons from a semi-infinite periodic potential. The semi-infinite magnonic crystal and the recurrent method are schematically illustrated in Fig. 3. The magnetic properties of one period of the semi-infinite magnonic crystal, including the profiles of the transition layers, are the same as in the case of the infinite magnonic crystal considered in the previous sections. The uniform ferromagnetic medium has anisotropy value of β_- while the rest of its parameters are equal to those of the magnonic crystal.

The amplitudes μ_0 and μ_r of the incident and reflected spin waves, respectively, are connected as

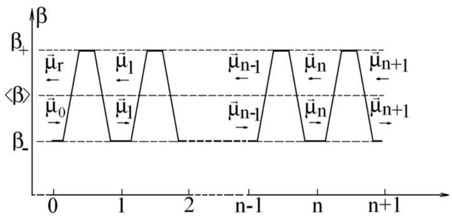

FIG. 3. The recurrent method used for calculation of the reflection coefficient is illustrated for the case of the semi-infinite magnonic crystal with a linear profile of the interface. The incident and reflected spin waves are schematically shown by forward and reverse arrows.

$$\mu_r = R\mu_0,$$

where R is the coefficient of reflection of spin waves from the semi-infinite magnonic crystal. The same relation will hold for spin waves $\vec{\mu}_n$ and $\overleftarrow{\mu}_n$ incident on and reflected from period n

$$\overleftarrow{\mu}_n = R\vec{\mu}_n. \tag{10}$$

Using recurrent relation

$$\vec{\mu}_n = \tau\vec{\mu}_{n-1} + \rho\overleftarrow{\mu}_n \tag{11}$$

we obtain

$$\frac{\vec{\mu}_n}{\vec{\mu}_{n-1}} = \frac{\tau}{1-\rho R},$$

where ρ and τ are the reflection and transmission coefficients through a single isolated symmetric period of the magnonic crystal.

In a similar way, we also obtain

$$\overleftarrow{\mu}_{n-1} = R\vec{\mu}_{n-1} = \rho\vec{\mu}_{n-1} + \tau\overleftarrow{\mu}_n$$

and hence

$$R = \rho + \frac{\tau^2 R}{(1-\rho R)}$$

According to Ref. 57, the solution of this equation is

$$R = \frac{\sqrt{(\rho+1)^2-\tau^2} - \sqrt{(\rho-1)^2-\tau^2}}{\sqrt{(\rho+1)^2-\tau^2} + \sqrt{(\rho-1)^2-\tau^2}}. \tag{12}$$

Here, ρ and τ are the reflection and transmission coefficients of spherical waves for a single isolated period of the magnonic crystal. Using the method of transfer matrices, one can obtain explicit expressions for ρ and τ as

$$\rho = \sqrt{1-4\tilde{M}^2}, \tag{13}$$

$$\tau = -2\tilde{M}. \tag{14}$$

Using the same method, it is possible to express the quasiwave number in terms of the coefficients ρ and τ.[57] The typical frequency dependence of the squared reflection coefficient R is shown in Fig. 4 for linear and sinusoidal profiles of the interface.

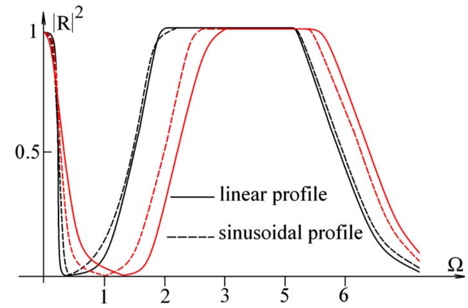

FIG. 4. (Color online) The frequency dependence of the squared reflection coefficient is shown for a linear and sinusoidal profiles of the interface for $\Delta\beta=2$, $\beta_-=3.0$, $h=0$, and $\delta_2=\delta_4=\delta$. The solid and dotted lines correspond to magnonic crystals with the linear and sinusoidal interfaces, respectively, with thicknesses of interfaces equal to $\delta/L=0.01$ (thin black lines) and $\delta/L=0.45$ (thick red lines).

V. RESULTS AND DISCUSSION

We begin by discussing general features of the spectra and scattering of spin waves in the studied magnonic crystals. As one could see already from Fig. 2, the size of magnonic band gaps depends markedly upon the thickness and profile of interfaces. The dependence of the size of the three lowest-lying magnonic band gaps upon the thickness of interfaces is plotted in Fig. 5 for both linear and sinusoidal profiles. Despite a sizable quantitative difference, the character of the dependence for the different interface profiles is qualitatively similar. So, the size of the band gaps first increases and then decreases as the thickness of the interface increases. In the limit of infinitely thin interfaces, the size of the band gaps in magnonic crystals with linear and sinusoidal interface profiles becomes equal, as expected. At the same time, it is important to note that the smeared interfaces do not actually lead to disappearance of magnonic band gaps in the spectrum. Thus, from the point of view of magnonic applications, such samples should not be considered as "faulty."

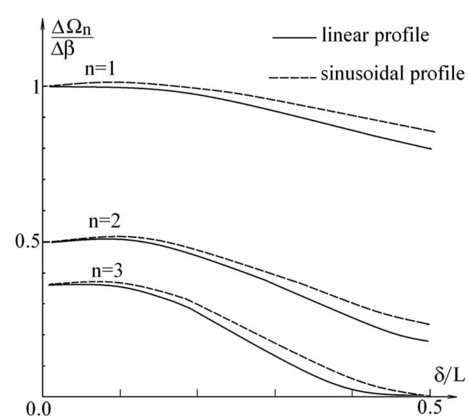

FIG. 5. The dependence of the size $\Delta\Omega_n$ of the three lowest-lying magnonic band gaps upon the thickness of interfaces is plotted for linear (solid line) and sinusoidal (dash-dotted line) profiles of the interfaces and $\Delta\beta=2$, $\beta_-=3.0$, $h=0$, $d_1=d_3$, and $\delta_2=\delta_4=\delta$. Here, n denotes the number of the band gap with $n=1$ corresponding to the lowest lying one.

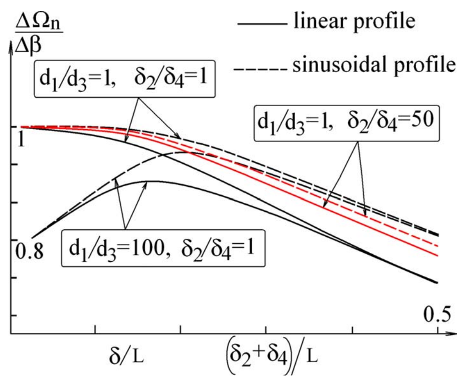

FIG. 6. (Color online) The dependence of the size of the lowest-lying magnonic band gap ($n=1$) upon the thickness of interfaces is plotted for linear (solid line) and sinusoidal (dash-dotted line) profiles of the interfaces, for two different ratios of the thicknesses of the main layers, and for two different ratios of the transition layers. $\Delta\beta=2$, $\beta_{-}=3.0$, $h=0$.

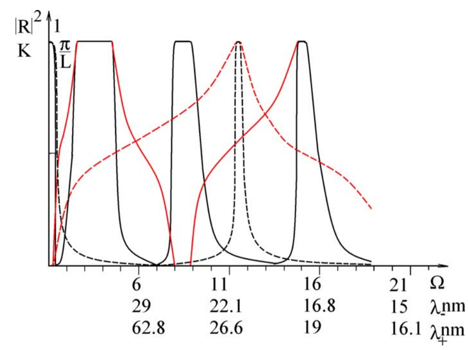

FIG. 7. (Color online) The frequency dependence of the squared reflection coefficient $|R|^2$ (black lines) is compared with the spectrum (red lines) for a magnonic crystal with linear profile and $\Delta\beta=2$ and $\delta/L=0.25$. The solid and dashed lines correspond to $L=600$ nm and $L=60$ nm, respectively.

Figure 6 shows the dependence of the size of the lowest-lying magnonic band gap ($n=1$) upon the thickness of interfaces for linear and sinusoidal interface profiles. One can see that, in the cases of "small" and "large" thicknesses of interfaces, the dominant role in determining the size of the band gaps is played by the relative thickness of the main layers and the structure of interfaces, respectively. Therefore, if the degree of smearing of interfaces is large, their thickness cannot be uniquely determined from spin-wave measurements but a particular functional form of the interface profile has to be assumed. From the applied point of view, the variation in the relative thicknesses of the main and/or transition layers creates an additional means by which to design magnonic crystal with a desired spectrum.

Generally, the size of the band gap decreases as the frequency at which it occurs increases. In the limit of small thicknesses of interfaces ($\delta\to 0$), which corresponds to the Kronig-Penny model, one obtains

$$\Delta\Omega_n = \frac{\Delta\beta}{n}\frac{d_1}{d_2} + o\left(\Delta\beta, \frac{d_1}{d_2}, n\right), \tag{15}$$

where $\Delta\Omega_n$ is the size of the nth band gap. This equation can be used to estimate the depth of modulation of the anisotropy value (or the relative thickness of the main layers) from measurements of the magnonic band gap, provided that the relative thickness of the main layers (or the depth of modulation of the anisotropy value $\Delta\beta$) is known.

In Fig. 7, we compare the frequency dependence of the squared reflection coefficient with the spectrum of spin waves. One readily notices that spin waves are fully reflected from the semi-infinite magnonic crystal at frequencies corresponding to band gaps in its spectrum, which occurs irrespective of the particular profile of the transition layers. As the period of the magnonic crystal decreases, the size of the band gaps also decreases. Within the allowed magnonic bands, there are points of zero reflection, corresponding to resonant transmission of spin waves through the magnonic crystal. The corresponding frequencies are likely to be working frequencies of magnonic devices incorporating both homogeneous and periodically modulated magnonic waveguides.

Let us discuss the range of realistic values of the geometrical parameters of such samples. In Ref. 52, the minimal thickness of nanowires is 10 nm, which is fairly close to the value of the exchange length in Co. The total length of the nanowires is 21 μm, which leads to a maximal period of 2 μm, assuming that the magnonic crystal contains at least ten periods. The smallest possible period is determined by the precision of the electrodeposition and can be as small as a few nanometers.[58] The thickness of interfaces can also be vanishingly small, if not atomically sharp, while the interface roughness could lead to a somewhat increased effective thickness of interfaces. At the same time, it might turn out to be more difficult to produce diffuse interfaces of large thickness since the magnetic phase of the deposited material is likely to vary discontinuously as function of the deposition parameters. Nonetheless, this should be more easily overcome in compositionally modulated nanowires. The roughness and more generally disorder can have a strong effect on the localization of magnonic modes in one dimension[41,55] and can strongly affect the size of band gaps.[38,39,44] In many experimental techniques, arrays of nanowires are preferred to isolated ones. In such arrays, the long-range magnetostatic interaction, the effect of which is roughly linear with respect to the packing fraction of the array,[59] is likely to modify the predictions of the present theory and even to lead to formation of a magnonic metamaterial.[60]

VI. SUMMARY

We have used the transfer-matrix method to solve the Landau-Lifshits equation and to develop a theory of exchange spin waves in a thin cylindrical magnonic crystal (periodically layered all-ferromagnetic nanowire) with diffuse interfaces. In particular, we have calculated the magnonic spectrum of the magnonic crystal and the reflection coefficient of spin waves from a junction between a homogeneous magnetic nanowire and the magnonic crystal. Solutions obtained for models in which the diffuse interfaces are represented by linear and sinusoidal profiles of variation in the uniaxial anisotropy value have been compared. An optional asymmetry in the relative thicknesses of either main layers, or interfaces, or both, has been taken into account and studied.

The major conclusion of the paper is that the thickness and profile of interfaces have a significant effect on the size and position of the magnonic band gaps but the interface smoothing does not generally lead to disappearance of the band gaps. In fact, the dependence of the magnonic spectrum and reflection coefficient on the profiles and relative thicknesses of interfaces might provide additional means by which to design magnonic crystals with a desired magnonic spectrum. Hence, our results may be useful for the emerging magnonic technology that will make use of propagating spin waves and modulated magnonic waveguides with the latter most likely having the "thin-nanowire" geometry considered here.

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APPENDIX

Here, we present details of the transfer-matrix formalism used in the calculations from the main text. The transfer matrices of the main (uniform) layers ($j=1,3$) are given by[56]

$$\hat{\mathbf{M}}_j = \begin{pmatrix} \cos(k_j d_j) & -k_j^{-1}\sin(k_j d_j) \\ k_j \sin(k_j d_j) & \cos(k_j d_j) \end{pmatrix}.$$

The transfer matrices of transition (nonuniform) layers ($j=2,4$) are obtained in a way similar to that in Refs. 47 and 48

$$\hat{\mathbf{M}}_j = \begin{pmatrix} P_j & Q_j \ell_j \\ -\tilde{P}_j \ell_j^{-1} & \tilde{Q}_j \end{pmatrix},$$

Here, for the linear profile, one has[47]

$$P_j = [\mathrm{Bi}'(0)\mathrm{Ai}(\zeta_j) - \mathrm{Ai}'(0)\mathrm{Bi}(\zeta_j)], \quad Q_j = [\mathrm{Bi}(0)\mathrm{Ai}(\zeta_j) - \mathrm{Ai}(0)\mathrm{Bi}(\zeta_j)],$$

$$\tilde{P}_j = [\mathrm{Bi}'(0)\mathrm{Ai}'(\zeta_j) - \mathrm{Ai}'(0)\mathrm{Bi}'(\zeta_j)], \quad \tilde{Q}_j = [\mathrm{Bi}(0)\mathrm{Ai}'(\zeta_j) - \mathrm{Ai}(0)\mathrm{Bi}'(\zeta_j)],$$

$$\zeta_j = -\frac{\ell_j^2}{\alpha}(\Omega - h - 2\pi - \beta_\pm), \quad \ell_j = (\delta_j \alpha/\Delta\beta)^{1/3},$$

where Ai and Bi are Airy functions,[61] and Ai′ and Bi′ are their derivatives.

For the sinusoidal profile, one has[48]

$$P_i = [\mathrm{ce}'(0)\mathrm{se}(\zeta_i) - \mathrm{se}'(0)\mathrm{ce}(\zeta_i)], \quad Q_i = [\mathrm{ce}(0)\mathrm{se}(\zeta_i) - \mathrm{se}(0)\mathrm{ce}(\zeta_i)],$$

$$\tilde{P}_i = [\mathrm{ce}'(0)\mathrm{se}'(\zeta_i) - \mathrm{se}'(0)\mathrm{ce}'(\zeta_i)], \quad \tilde{Q}_i = [\mathrm{ce}(0)\mathrm{se}'(\zeta_i) - \mathrm{se}(0)\mathrm{ce}'(\zeta_i)],$$

$$\zeta_j = 5\delta_j \Delta\beta/2\ell_j, \quad \ell_j = \sqrt{\frac{\alpha}{\Omega - h - \beta_\pm}},$$

where se and ce are Mathieu functions, and se′ and ce′ are their derivatives.

The transfer matrix of one period of the magnonic crystal can then be written as

$$\hat{\mathbf{M}} = \prod_{j=1}^{4} \hat{\mathbf{M}}_j.$$

Finally, the trace (spur) of the transfer matrix is

$$\begin{aligned}
\mathrm{Tr}(\hat{\mathbf{M}}) = {} & \{[\cos(k_1 d_1)P_2 + (\ell_2 k_1)^{-1}\sin(k_1 d_1)P_2']\cos(k_3 d_3) \\
& + [\cos(k_1 d_1)Q_2 \\
& - (\ell_2 k_1)^{-1}\sin(k_1 d_1)Q_2']\ell_4 k_3 \sin(k_3 d_3)\}P_4 \\
& + \{[\cos(k_1 d_1)Q_2 - (\ell_2 k_1)^{-1}\sin(k_1 d_1)Q_2']\cos(k_3 d_3) \\
& - [\cos(k_1 d_1)P_2 + (\ell_2 k_1)^{-1}\sin(k_1 d_1)P_2'] \\
& \times(\ell_4 k_3)^{-1}\sin(k_3 d_3)\}P_4' - \{[\ell_2 k_1 \sin(k_1 d_1)P_2 \\
& - \cos(k_1 d_1)P_2']\cos(k_3 d_3) + [\ell_2 k_1 \sin(k_1 d_1)Q_2 \\
& + \cos(k_1 d_1)Q_2']\ell_4 k_3 \sin(k_3 d_3)\}Q_4 \\
& + \{[\ell_2 k_1 \sin(k_1 d_1)Q_2 + \cos(k_1 d_1)Q_2']\cos(k_3 d_3) \\
& - [\ell_2 k_1 \sin(k_1 d_1)P_2 - \cos(k_1 d_1)P_2'] \\
& \times(\ell_4 k_3)^{-1}\sin(k_3 d_3)\}Q_4'.
\end{aligned}$$

This expression contains all structural information about the magnonic crystal and is used in Eqs. (9) and (12)–(14) to calculate the spin-wave dispersion in and reflection from the magnonic crystal, respectively.

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