Non-Appearance of Vortices in Fast Mechanical Expansions of Liquid ^{4}He Through the Lambda Transition

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A new experiment has been performed to study the formation of topological defects (quantized vortices) during rapid quenches of liquid ^{4}He through the superfluid transition, with particular care taken to minimise vortex creation via conventional hydrodynamic flow processes. It is found that the generated vortices, if any, are being produced at densities at least two orders of magnitude less than might be expected on the basis of the Kibble-Zurek mechanism.

11.27.+d, 05.70.Fh, 11.10.Wx, 67.40.Vs

When a physical system passes rapidly enough through a continuous phase-transition, creation of topological defects is to be expected [1] owing to the causal disconnection of separated regions. This idea was proposed by Kibble [2] in connection with the GUT (grand unified theory) symmetry-breaking phase-transition of the early universe, and later developed in detail by Zurek [3–5] who introduced simple physical arguments to estimate the density of defects in terms of the rate at which the system passes through the transition. Zurek also pointed out that the process of defect production should be generic, applicable in principle to all continuous phase-transitions, thus opening up the remarkable possibility of testing aspects of cosmological theories on the laboratory bench. The first experiments of this kind were performed on liquid crystals [6,7] using weakly first-order phase-transitions. Later, the corresponding experiments were carried out using the second-order superfluid phase-transitions of liquid ^{4}He [8] and liquid ^{3}He [9,10]. In all cases, the generated defect densities were reported as being consistent with Zurek's [3–5] estimates. In this Letter we describe an improved version of the ^{4}He experiment [8] in which particular care has been taken to minimise the production of quantized vortices – the relevant topological defects in liquid helium – through ordinary hydrodynamic flow processes. As we shall see, our new results show no convincing evidence of any vortex creation at all, but they enable us to place an approximate upper bound on the initial density of vortices produced via the Kibble-Zurek mechanism.

The lambda (superfluid) transition in liquid ^{4}He provided the basis of Zurek's original proposal [3] for a cosmological experiment. The underlying idea is very simple. A small isolated volume of normal (non-superfluid) liquid ^{4}He is held under pressure, just above the temperature T_λ of the lambda transition. Although the logarithmic infinity in its heat capacity at T_λ makes it impossible to *cool* the sample quickly into the superfluid phase, Zurek pointed out that the pressure dependence of T_λ meant that *expansion* of the sample would allow it

to be taken through the transition very rapidly. Thus, closely paralleling the processes through which topological defects (e.g. cosmic strings [12]) are believed to have been produced in the GUT phase-transition 10^{-35} s after the big bang, huge densities of quantized vortices [11] should be produced at the transition. The "bulk version" [4] of this experiment was subsequently realised [8] in a specially designed expansion apparatus [13] in which second-sound attenuation was used to probe the decaying tangle of vortices created during the expansion. The initial vortex density, obtained by back-extrapolation to the moment ($t = 0$) of traversing the transition, was $\sim 10^{12}$–10^{13} m^{-2}, consistent with the theoretical expectation [3–5].

An unexpected observation in the initial experiments [8] was that small densities of vortices were created even for expansions that occurred wholly in the superfluid phase, provided that the starting point was very close to T_λ. The phenomenon was initially [13] attributed to vortices produced in thermal fluctuations within the critical regime, but it was pointed out [14] that effects of this kind are only to be expected for expansions starting within a few μK of the transition, i.e. much closer than the typical experimental value of a few mK. The most plausible interpretation – that the vortices in question were of conventional hydrodynamic origin, arising from nonidealities in the design of the expansion chamber – was disturbing, because expansions starting above T_λ traverse the same region. Thus some, at least, of the vortices seen in expansions through the transition were probably not attributable to the Zurek-Kibble mechanism as had been assumed. It has been of particular importance, therefore, to undertake a new experiment with as many as possible of the nonidealities in the original design eliminated or minimised.

An ideal experiment would be designed so as to avoid all fluid flow parallel to surfaces during the expansion. This could in principle be accomplished by e.g. the radial expansion of a spherical volume, or the axial expansion of a cylinder with stretchy walls. In either of these cases,

the expansion would cause no relative motion of fluid and walls in the direction parallel to the walls and presumably, therefore, no hydrodynamic production of vortices. The walls of the actual expansion chamber [8,13] were made from bronze bellows, thus approximating the cylinder with stretchy walls. Although there must, of course, be some flow parallel to surfaces because of the convolutions, such effects are relatively small. It is believed that the significant nonidealities, in order of importance, arose from: (a) expansion of liquid from the filling capillary, which was closed by a needle valve 0.5 m away from the cell; (b) expansion from the shorter capillary connecting the cell to a Straty-Adams capacitive pressure gauge; (c) flow past the fixed yoke on which the second-sound transducers were mounted. In addition (d) there were complications caused by the expansion system bouncing against the mechanical stop at room temperature.

The new expansion cell, designed to avoid or minimise these problems, is sketched in Fig. 1. The main changes from the original design are as follows: (a) the sample filling capillary is now closed off at the cell itself, using a hydraulically-operated needle-valve; (b) the connecting tube to the pressure gauge has been eliminated by making its flexible diaphragm part of the chamber end-plate; (c) the second-sound transducers are also mounted flush with the end-plates of the cell, eliminating any support structure within the liquid; (d) some damping of the expansion was provided by the addition of a (light motor vehicle) hydraulic shock-absorber.

The operation of the expansion apparatus and the technique of data collection/analysis were much as described previously [8,13], except that the rate at which the sample passed through the lambda transition was determined directly by simultaneous measurements of the position of the pull-rod (giving the volume of the cell, and hence its pressure) and the temperature of the cell. Distance from the transition was defined in terms of the parameter

$$\epsilon = \frac{T_\lambda - T}{T_\lambda} \tag{1}$$

Thus the pressure-dependence of T_λ, and the nonconstant expansion rate, were taken explicitly into account. Part of a trajectory recorded close to the transition during a typical expansion is shown in Fig. 2. From such results, the quench time

$$\tau_Q = \frac{1}{\left(\frac{d\epsilon}{dt}\right)_{\epsilon=0}} \tag{2}$$

is easily determined. In the case illustrated, it was $\tau_Q = 17\pm1.0$ ms. The position of the pull-rod is measured with a ferrite magnet attached to the rod, moving within a coil fixed to the cryostat top-plate. There is some evidence of a small time-lag between the expansion of the cell under the influence of the high-pressure liquid inside it, and the corresponding movement of the transducer. Thus the measured τ_Q may tend to overestimate the quench time slightly.

Following an expansion though the transition, after T, P and the velocity c_2 of second sound have settled at constant values, a sequence of second-sound pulses is propagated through the liquid. If the anticipated tangle of vortices is present, the signal may be expected to grow towards its vortex-free value as the tangle decays and the attenuation decreases. Signal amplitudes measured just after two such expansions are shown by the data points of Fig. 3. It is immediately evident that, unlike the results obtained from the original cell [8], there is now no evidence of any systematic growth of the signals with time or, correspondingly, for the creation of any vortices at the transition. One possible reason is that the density of vortices created is smaller than the theoretical estimates [3–5], but we must also consider the possibility that they are decaying faster than they can be measured: there is a "dead period" of about 50 ms after the mechanical shock of the expansion, during which the resultant vibrations cause the signals to be extremely noisy (which is why the error bars are large on early signals in Fig. 3).

The rate at which a tangle of vortices decays in this temperature range is determined by the Vinen [15] equation

$$\frac{dL}{dt} = -\chi_2 \frac{\hbar}{m_4} L^2 \tag{3}$$

where L is the length of vortex line per unit volume, m_4 is the ^{4}He atomic mass and χ_2 is a dimensionless parameter. The relationship between vortex line density and second-sound attenuation is known [11] from experiments on rotating helium, and may for present purposes be written in the form

$$L = \frac{6c_2 \ln(S_0/S)}{B\kappa d} \tag{4}$$

where S_0 and S are the signal amplitudes without and with vortices present, B is a temperature dependent parameter, $\kappa = h/m_4$ is the quantum of circulation, and d is the transducer separation.

Integrating (3) and inserting (4), one finds immediately that the recovery of the signal should be of the form

$$\left[\ln\left(\frac{S_0}{S}\right)\right]^{-1} = \frac{6c_2}{\kappa B d}\left(\chi_2 \frac{\kappa}{2\pi} t + L_i^{-1}\right) \tag{5}$$

Of the constants in (5), all are known except χ_2 and B which seem not to have been measured accurately within the temperature range of interest. We therefore performed a subsidiary experiment, deliberately creating vortices by conventional means and then following their decay by measurements of the recovery of the second-sound signal amplitude. This was accomplished by leaving the needle-valve open, so that ~ 0.2 cm^3 of liquid

from the dead volume outside the needle-valve actuator-bellows was forcibly squirted into the cell during an expansion. As expected, large densities of vortices were created. Plots of $[\ln(S/S_0)]^{-1}$ as a function of t yielded straight lines within experimental error in accordance with (5), as shown in Fig. 4. By measurement of the gradient, and comparison with (5), for a number of such plots it was possible to determine χ_2/B as a function of T and P. Fuller details willl be given elsewhere, but it was found that χ_2/B did not exhibit a strong temperature dependence and could be approximated by $\chi_2/B = 0.004 \pm 0.001$ within the range of interest where $0.02 < \epsilon < 0.06$.

The measured value of χ_2/B was then used to calculate the evolution of S/S_0 with time for different values of L_i, yielding the curves shown in Fig. 3. From the τ_Q measured from the gradient in Fig. 2, and Zurek's estimate of

$$L_i = \frac{1.2 \times 10^{12}}{(\tau_Q/100\,\mathrm{ms})^{2/3}} \qquad [\mathrm{m}^{-2}] \qquad (6)$$

we are led to expect that $L_i \approx 4 \times 10^{12}$ m^{-2}. A comparison of the calculated curves and measured data in Fig. 3 shows that this is plainly not the case. In fact, the data suggest that L_i is smaller than the expected value by at least two orders of magnitude.

There are a number of comments to make on this null result which, in the light of the apparently positive outcome from the earlier experiment [8], came as a considerable surprise. First, Zurek's estimates of L_i were never expected to be accurate to better than one, or perhaps two, orders of magnitude, and his more recent estimate [16] suggests somewhat lower defect densities. So it remains possible that his picture [3–5,16] is correct in all essential details, and that the improved experiment with faster expansions now being planned will reveal evidence of the Kibble-Zurek mechanism in action in liquid ^{4}He. Secondly, it must be borne in mind that (3), and the value of χ/B measured from the data of Fig. 4, refer to hydrodynamically generated vortex lines. Vorticity generated in the nonequilibrium phase transition may be significantly different, e.g. in respect of its loop-size distribution [17], may therefore decay faster, and might consequently be unobservable in the present experiments. Thirdly however, it seems surprising that the ^{3}He experiments [9,10] appear to give good agreement with Zurek's original estimates [3–5] whereas the present experiment shows that they overestimate L_i by at least two orders of magnitude. It is not yet known for sure why this should be, but an interesting explanation of the apparent discrepancy is being proposed [18] by Karra and Rivers.

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FIG. 1. Sketch of the improved expansion cell.

FIG. 2. Part of a typical quench trajectory, showing the time evolution of ϵ, the parameter (1) specifying distance below the lambda transition. The zero of time t is taken to be the instant when $\epsilon = 0$. The curve is a guide to the eye, and its slope at $t = 0$ enables the quench time τ_Q to be determined from (2).

FIG. 3. Evolution of the second-sound amplitude S with time, following an expansion of the cell at $t = 0$ (data points), normalised by its vortex-free value S_0. The open and closed symbols correspond to signal repetition rates of 100 and 50 Hz respectively, and in each case the starting and finishing conditions were $\epsilon_i = -0.032$, $\epsilon_f = 0.039$. The curves refer to calculated signal evolutions for different initial line densities, from the bottom, of $10^{12}, 10^{11}$ and 10^{10} m^{-2}.

FIG. 4. Evolution of the second-sound amplitude S following an expansion of the cell with the needle-valve left *open*, thereby causing hydrodynamic vortex creation. The data are plotted so as to yield a straight line on the basis of (5).

pressure gauge diaphragm

phosphor-bronze bellows

orifice and needle

sample entry port

heater

^{4}He sample

bolometer

pressurised liquid ^{4}He to close needle valve