Interactions between Sheets of Phonons in Liquid ^{4}He

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Abstract

We have created two sheets of $\sim$1 K phonons in liquid ^{4}He at $\sim$55 mK such that they intersect each other as they move towards a common point. If the two sheets have a small angle between them, they interact strongly and create a hot line in the liquid helium. This line is continuously fed with energy from the two sheets and loses energy by creating high energy phonons. If the angle between the sheets is larger than $\sim 30°$ they do not interact but pass through each other. These results give direct evidence for the composition of the sheets: they comprise strongly interacting low-energy phonons which occupy a narrow cone in momentum space.

Phonon pulses have been created in cold liquid ^{4}He over many years since the initial work of Gernsey and Luszczynski in 1971 [1]. The pulses were found to travel at the velocity of sound and so it was believed, that to first order, the phonons were independent, non-interacting ballistic wave packets. The liquid ^{4}He was sufficiently cold that the ambient thermal phonons could be ignored. However it was recognised that phonons could spontaneously decay by the three phonon process (3pp) [2, 3], which is allowed by the upward curvature of the dispersion curve [4, 5]. This picture was challenged recently by Adamenko et al. [6, 7] who argued that a number of observations, such as the creation of high energy phonons in the liquid, could be explained if the phonons in the pulse were treated as strongly interacting and were confined to a narrow cone in momentum space. This has prompted us to look for evidence of the proposed phonon momentum distribution.

A short electrical pulse in a planar thin-film heater is usually used to create a pulse of phonons. It was proposed [6, 7], that the phonons in the

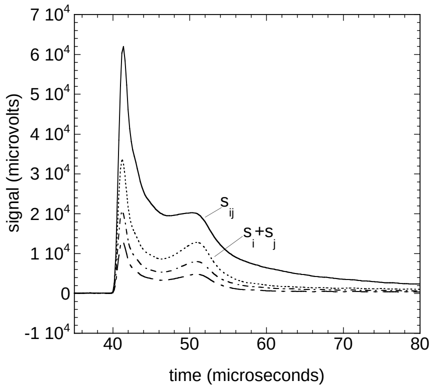

Figure 1: This shows the bolometer signals for the two separate pulses, dashed-dot lines, and for the double pulse, solid line. the sharp peak at 40 μs is due to the l-phonons and the signal after $\approx 43\mu$s is due to h-phonons. The sum of the signals from the two separate pulses is shown as the dotted line, it can be seen to be about half that of the double pulse. The input pulses are 100ns and 6.3mW

pulse interact by the three phonon process (3pp) [8] which is very rapid and creates a thermal equilibrium with temperature $T{\sim}1$ K, within the cone in momentum space, in a time of order of 10^{-10} s [9]. The cone is thought to arise because only phonons with a small angle $\theta_{3\mathrm{pp}}$, typically 10° between the two momentum vectors, interact by the 3pp. This causes the magnitude of the cone angle to be $\sim \theta_{3\mathrm{pp}}$.

There is already some indirect evidence for this picture: the creation of high energy phonons, (h-phonons) with energy ϵ >10K can be explained by four phonon interactions between the low energy phonons (l-phonons) within the pulse. Several properties of these h-phonons have been measured [10, 11, 12] and modelled theoretically [6, 7, 13]. Recently, it has been found experimentally that the temperature of the phonon distribution is uniform in the centre and decreases towards the edges of the pulse [14] and this behaviour can also be explained within this picture [15].

Despite this indirect evidence for the properties of the phonon pulse, there has been no previous direct confirmation of them. In this paper we present remarkable experimental results which provide some compelling evidence that the phonons are strongly interacting and occupy a narrow cone in momentum space.

In figure 1 we show the signals from two heaters which are separated by an angle $\alpha = 6°$. Signals from separate and simultaneous pulses to the two heaters are shown. The dotted line is the sum of the two separate pulses. All

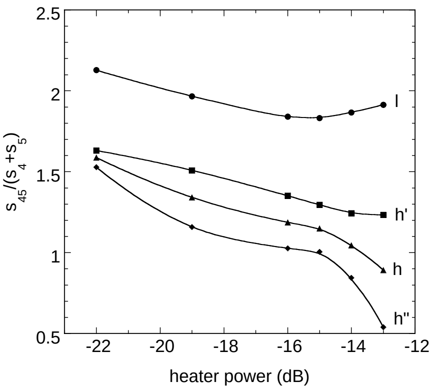

Figure 2: For heaters with an angle between them of 6°, the ratio of the integrated signals from the double pulse to the sum of the integrated signals from the separate pulses, is shown as a function of heater power (the reference power is 0.5W, so -22, -19, -16 and -13dB correspond to 3.1, 6.3, 12.5 and 25mW respectively). The curves labelled l, h', h and h" are integrated over time intervals 40-43, 43-51.6, 43-73 and 51.6-73μs respectively. The pulse durations are 100ns.

pulses are 100ns and 6.3 mW (-19dB). It can be seen that the fast l-phonon peak, from simultaneous pulses, is twice as high as that of the sum of the two separate pulses. This is perhaps surprising, as interactions usually scatter phonons out of the beam and so reduce the signal. The slower h-phonon signal, from the simultaneous pulses, is also substantially larger than the sum of the separate pulses.

To quantify these increases we have integrated the signals between 41.0 and 44.0 μs for the l-phonons and between 44.0 and 73 μs for the h-phonons. We divide this range at 51.6 μs to distinguish the faster h-phonon signal (44.0 - 51.6 μs) from the slower one (51.6 - 73 μs). 51.6 μs is the propagation time for a phonon with energy $\epsilon = 10$ K (the fastest h-phonon) to go from the heater to the bolometer. The time 73 μs is chosen to cover most of the h-phonon signal without including the low level signal which comes from the slow cooling of the glass heater substrate, after the heater pulse.

We define the ratio $R = S_{ij}/(S_i + S_j)$ which is the fractional increase of the integral of the the double pulse (S_{ij}) over the sum of the integrals of the two separate pulses (S_i and S_j), see figure 1, during the same time interval. The ratio is equal to 1 if there is no interaction. This ratio is shown as a function of heater power in figure 2. We see that for the l-phonons $R \sim 2$ and for h-phonons $R \sim 1.3$ but is substantially larger at lower power ($R = 1.6$) than at high power ($R = 0.9$). We see too that the ratio for the faster h-phonons is higher than for the slower ones. The results for 50 ns pulses are broadly similar to those for 100ns.

The two main features of these results that need explaining are: (a), where the extra detected energy comes from and (b) why the extra h-phonon signal arrives earlier than for single pulses. As the double-pulse energy is equal to the sum of the energies in the two separate pulses, the increased detected signal with the double pulse must be due to a concentration of the injected energy onto the bolometer. We shall now describe how this occurs.

A short heater pulse, $t_{\mathrm{p}} = 100$ ns, creates a phonon sheet. The thickness of the sheet is ct_p where c is the velocity of sound; $ct_p = 238 \times 10^{-7} = 24$ μm. At a distance from the heater, the width of the sheet is somewhat larger than the heater width which is 1 mm $\times$ 1 mm. After 17 mm the sheet is $\approx$ 3 mm wide [14]. For $<$10 mm we estimate the sheet dimensions to be 2 mm $\times$ 2 mm $\times$ 24 μm, so a sheet is an accurate description.

Figure 3 illustrates the essential elements of the phonon sheets. Figure 3a shows a schematic picture of the phonon sheet created by a single heater. The momentum of the phonons in the sheet, are in a cone with angle $\sim \theta_{\mathrm{3pp}}$, as shown in figure 3b. The phonons have a temperature which falls in the range $\sim$ 1.0 to $\sim$ 0.7 K: as they propagate they cool due to the creation of h-phonons [7] and also due to transverse expansion of the sheet [15].

Our experiment probes one l-phonon sheet with another similar sheet. The sheets are at a small angle α to each other as shown in figure 3c. The effect of the interaction between the sheets is remarkable. Along the common line of the two sheets a hot line of phonons is created. It will be noticed that the sheets do not extend beyond this hot line because of the strong interactions along the line of overlap of the two sheets. This process concentrates the energy in the sheets as they move forward. In the absense of interactions, each sheet would extend behind the other sheet, as shown in figure 3e.

The two cones in momentum space overlap, this is shown hatched in figure 3d. The phonons in the overlap volume rapidly thermalise within a time of order 10^{-10} s. This creates a new momentum cone, with angle $\sim \theta_{\mathrm{3pp}}$, associated with the hot line, with its axis along the bisecting line of the cone axes from the two sheets.

In figure 3e, we show two sheets with a large angle between them. In this situation there is no interaction and the two sheets just pass through each other. The two momentum cones, shown in figure 3f, do not overlap. The narrow angle of the cone in momentum space will be confirmed by showing that the two sheets do not interact if the angle between the sheets is too large.

For the experiment, we arranged a set of heaters on an arc of a circle, radius 10 mm, and a detector at the centre of the circle. The heaters are

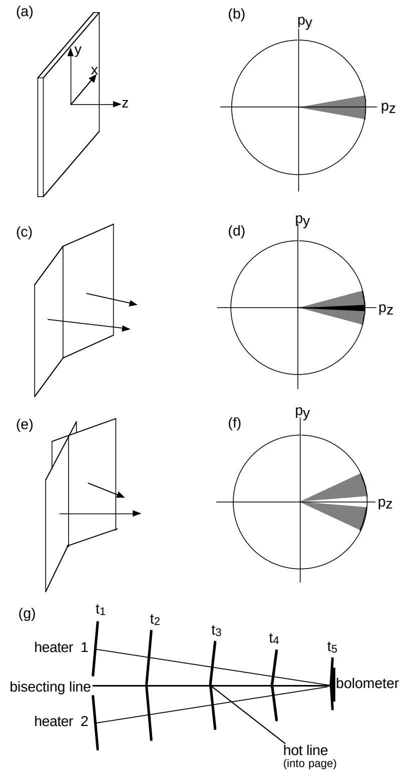

Figure 3: (a) shows schematically a phonon sheet moving in the z direction. (b) shows the cone in momentum space occupied by the phonons in the sheet. (c) shows two phonon sheets at a small angle to each other, the normals to the two sheets are indicated. (d) the overlap of the two cones in momentum space is shown. (e) shows two phonon sheets with a large angle between them. Note that the sheets pass through each other. (f) shows the two occupied cones separated. (g) shows schematically a plan view of the two phonon sheets at different times. The sheets are created at time t_1 and at t_2 they start to touch and begin to create the hot line. For $t > t_2$ the area of the sheets decrease and their energy is fed into the hot line. The h-phonons, created by the hot line, trail behind the two phonon sheets, but are not shown.

1 mm $\times$ 1 mm gold films on glass cover slips. Their resistance is $\sim 50\Omega$. They can be pulsed individually or in pairs. The pulses are 50 or 100 ns duration with powers in the range 3.1 mW (-22 dB) to 25 mW (-13 dB). The superconducting zinc film detector is cut into a serpentine track to give an operating resistance of $\sim$50Ω. It is held at a constant temperature by an electronic feedback signal which self-heats the zinc and indicates the phonon energy flux, [16, 17, 18]. The dominant time constant of the detector system is $\approx$1.5 μs. Many pulses are averaged together to increase the signal to noise ratio using a Tektronix DSA 601A. The helium is isotopically pure [19] and at a temperature $\sim$55 mK.

Figure 3g shows a plan view of the spatial positions of the two phonon sheets at a series of times. At time t_1 the sheets are just formed at the heater. At time t_2 the edges of the sheets just touch. This overlap of the sheets starts the creation of a line of higher density phonons, a hot line, along their common line. This is the beginning of an extraordinary process: the interactions between the phonons in the sheets are so strong that the interacting phonons are assimilated into a new population with its own momentum cone, with its axis along the bisecting line. For $t > t_2$ the hot line moves forward along the bisecting line and the sheets get smaller as their energy is fed into the hot line.

The hot line does not continuously get hotter as it quickly reaches a temperature where the rate of energy loss due to the creation of h-phonons equals the rate of gain of energy from the two sheets. h-phonons are created all the way from the position at t_2 to the bolometer. It is this property which accounts for the fast h-phonon signal; as the hot line moves with velocity c, so h-phonons created near the bolometer arrive just after the l-phonon signal from the two sheets. This sustained source of h-phonons contrasts with the h-phonons produced by a single sheet. In this case, the sheet cools as it moves from the heater towards the bolometer so the h-phonon production decreases with distance from the heater . This makes the average h-phonon signal slower than with the hot line; this is clearly visible in figure 1.

The higher density of l-phonons in the hot line causes the increase in the l-phonon signal. The measurements show that there is as much energy from the hot line as in the area of the sheets that are separately incident on the bolometer. In separate pulses, the sheets are wider than the bolometer so only a fraction of each phonon sheet is incident on the bolometer. In a double pulse some of this energy, that would otherwise miss the bolometer, is collected by the hot line. If the phonon sheet is 2mm $\times$ 2mm then the energy in an extra 0.5 mm^2 area of each sheet falls on the bolometer which has an area of 1 mm^2. This gives a 50% increase in signal. This fraction is independent of the angle α between the sheets so long as $\alpha < \theta_{3pp}$ and

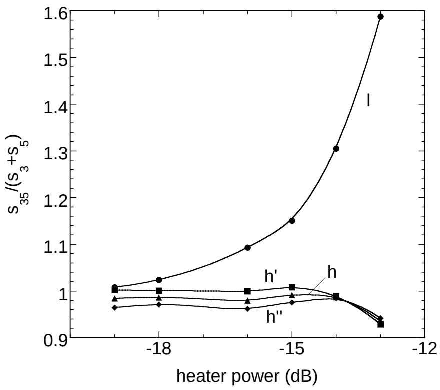

Figure 4: For heaters with an angle α between them of 26°, the ratio R of the integrated signals from the double pulse to the sum of the integrated signals from the separate pulses, is shown as a function of heater power (the reference power is 0.5 W, so -22, -19, -16 and -13 dB correspond to 3.1, 6.3, 12.5 and 25 mW respectively). The curves labelled l, h' ,h, h" are integrated over time intervals 40-43, 43-51.6, 43-73 and 51.6-73 μs respectively. The pulse durations are 100 ns.

the bolometer is at the centre of curvature. This simple geometric argument leaves out the effects of the expansion of the phonon sheet and the hot line. The transverse expansion of the phonon sheet means the separate l-phonon signals are smaller but the hot line still collects up the same fraction of the sheets so the ratio $S_{ij}/(S_i+S_j)$ will be larger. The estimate thus agrees with the measurements.

An important feature of our picture is that the phonons in the sheet are in a narrow cone in momentum space. This can be tested by varying the angle between the two sheets as there should be no interactions between them when the two cones in momentum space do not overlap. We define θ_{3pp}, in the 3pp scattering $l_1 \leftrightarrow l_2 + l_3$, as the angle between l_2 and l_1 when l_2 and l_3 have the same energy $\epsilon_1/2$. For a phonon distribution at temperature T, 0.8 of the phonons have energy $\epsilon <$ 4T so we use this energy to evaluate θ_{3pp}. So for example, using the measurements of the upward dispersion [21], when $T = 1$ K, $\epsilon_1 = 4$ K and $\theta_{3\mathrm{pp}} = 9.7°$

In reality, the cones do not have a sharp boundary because there are 3pp interactions where $\epsilon_2 >> \epsilon_3$ which have a larger angle between them. In the example above, the maximum included angle between l_2 and l_3 is 20°, but the probability of this process is much lower than for $\epsilon_2 = \epsilon_3$. These angles scale roughly with T for $T <$ 1K. The phonons in the high energy tail of the distribution can have somewhat larger angles; the maximum θ_{3pp}=12° and the maximum included angle is 24°.

Significant overlap of the cones requires $\alpha < 2\theta_{3pp}$. This is confirmed by the two heaters with $\alpha = 46°$ where we find no interaction at all: the signals from double pulses are equal to the sum of the signals from two separate pulses.

When the angle between the two heaters is $\alpha = 26°$ the situation is marginal. With low power pulses, 100 μs, 6.3 mW (-19 dB), there is little interaction, but for higher powers, 12.5 (-16 dB) and 25 mW (-13 dB) there is an increasing interaction shown by the ratio $R = S_{ij} \ / \ (S_i + S_j) > 1$. We show this in figure 4 which is in stark contrast to figure 2. When $\alpha = 26°$ only the value of R for the l-phonons shows an increase; the h-phonons have $R \sim 1$ and even $R < 1$. It is, of course, much easier to get an increase in the l-phonon ratio than the h-phonon ratio because the l-phonon signal can be increased by an interaction between the sheets just in front of the bolometer. In contrast, the h-phonon signal depends on interactions occuring over a substantial fraction of the flight path.

In conclusion, we have shown that l-phonon sheets interact when the angle between them is small, but not when it is large. This is direct evidence that the phonons in the sheet occupy a narrow cone in momentum space with a cone angle, typically $\sim 10°$. When the angle between the sheets is small, the interaction between the sheets is strong and leads to a hot line in the liquid helium. This line moves at the velocity c=238 m s^{-1} along the bisecting line, between the two normals to the heaters. The hot line is continuously fed with energy from the decreasing area of the sheets, and it loses energy by creating h-phonons. The hot line will be in dynamic equilibrium and at an essentially constant temperature over its path. The creation of the hot line collects energy from the sheets which would otherwise miss the bolometer. This leads to higher l- and h-phonon signals than the sum of the two separate signals.

We see this behaviour when the angle between the sheets is 6°. When the angle between the sheets is 26°, there are interactions when the pulse power is high, but not when it is low. In this marginal case, the slightly larger width of the cone at higher powers can initiate interactions which lead to increasingly stronger interactions. When the angle is 46° we find that there are no interactions.

This striking behaviour can be understood if the phonons in the sheets are both in a narrow momentum cone and are strongly interacting. This is the most direct evidence to date for the properties of the phonon sheet that has been postulated and used to explain several diverse experimental results.

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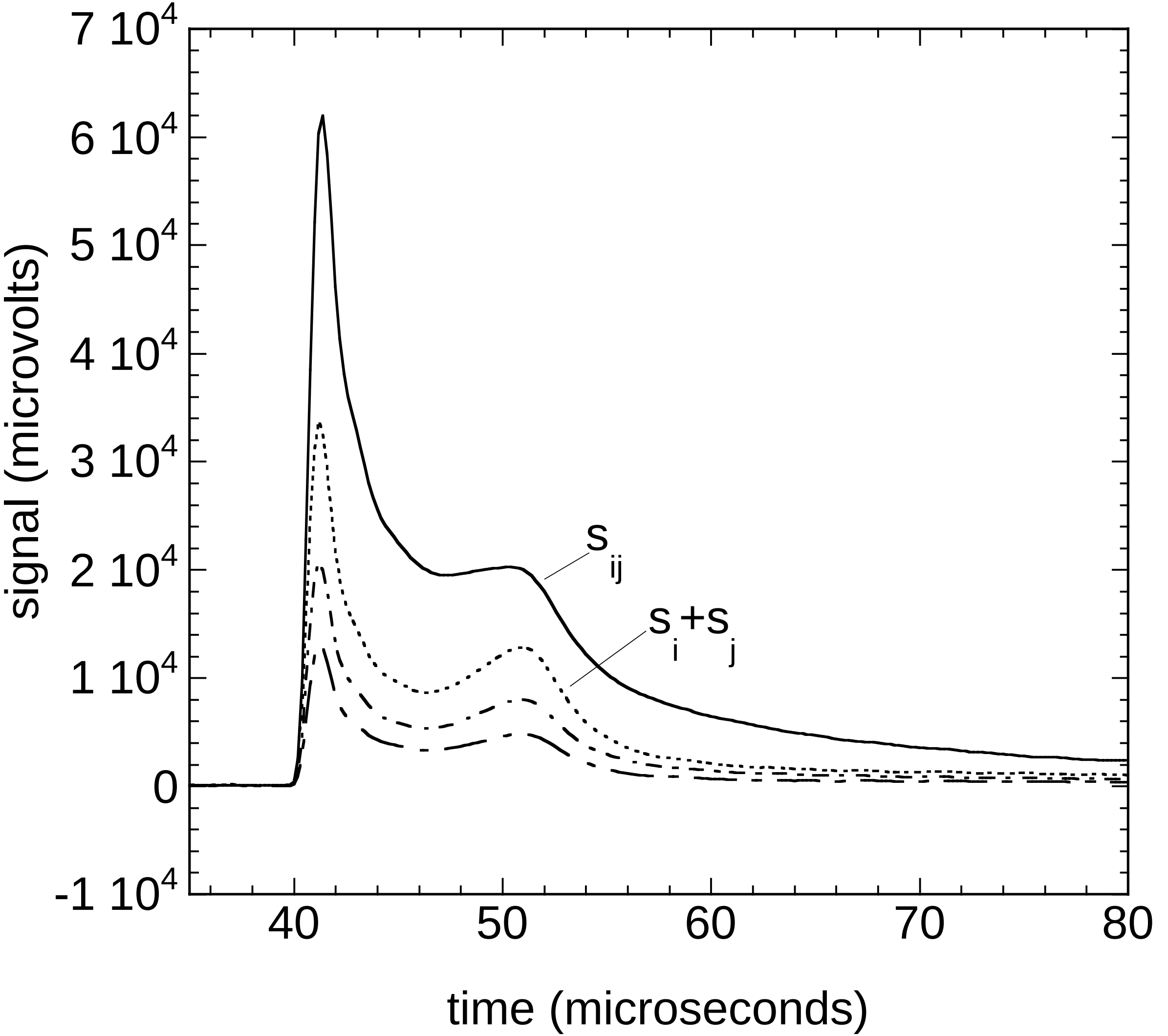

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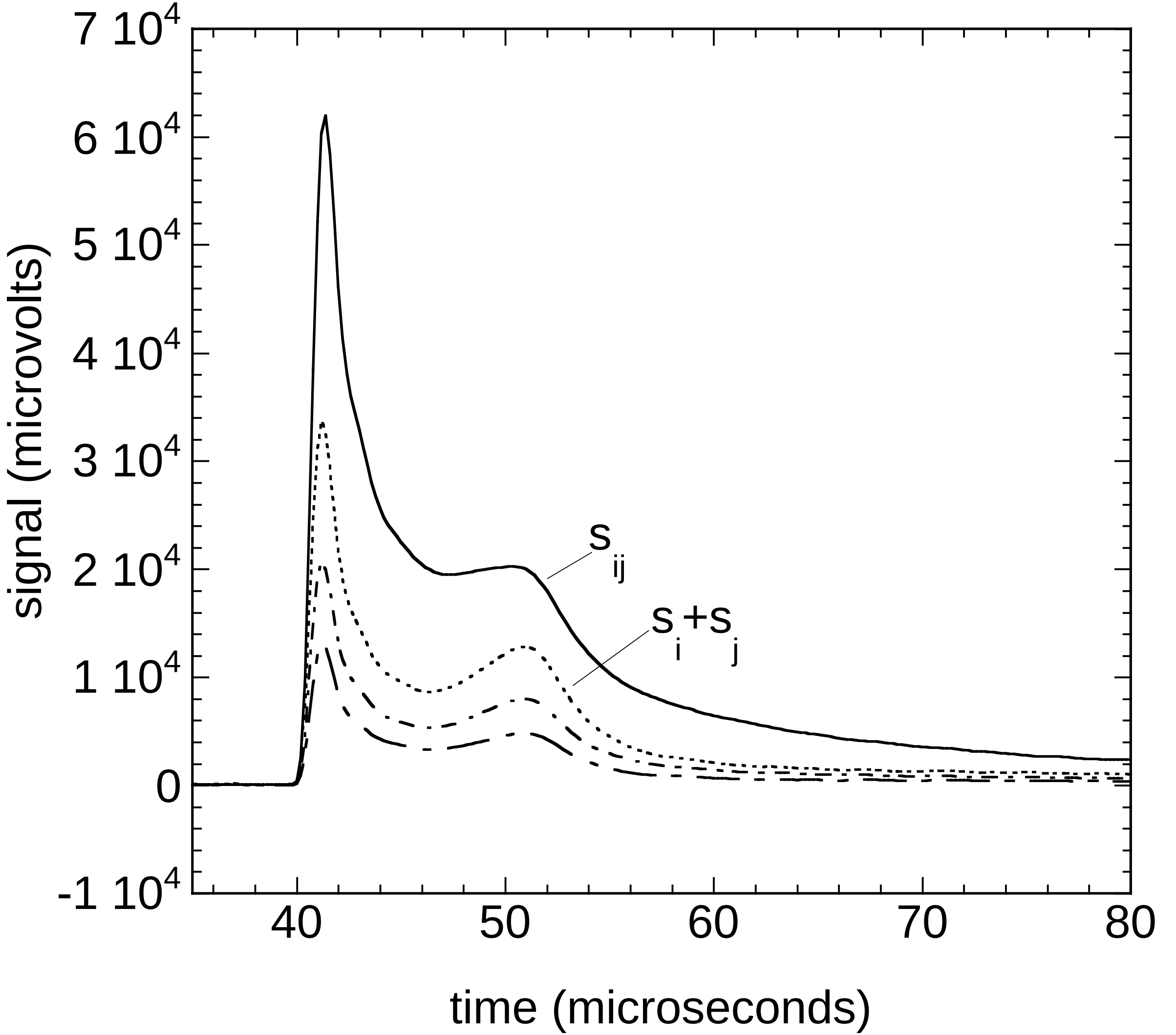

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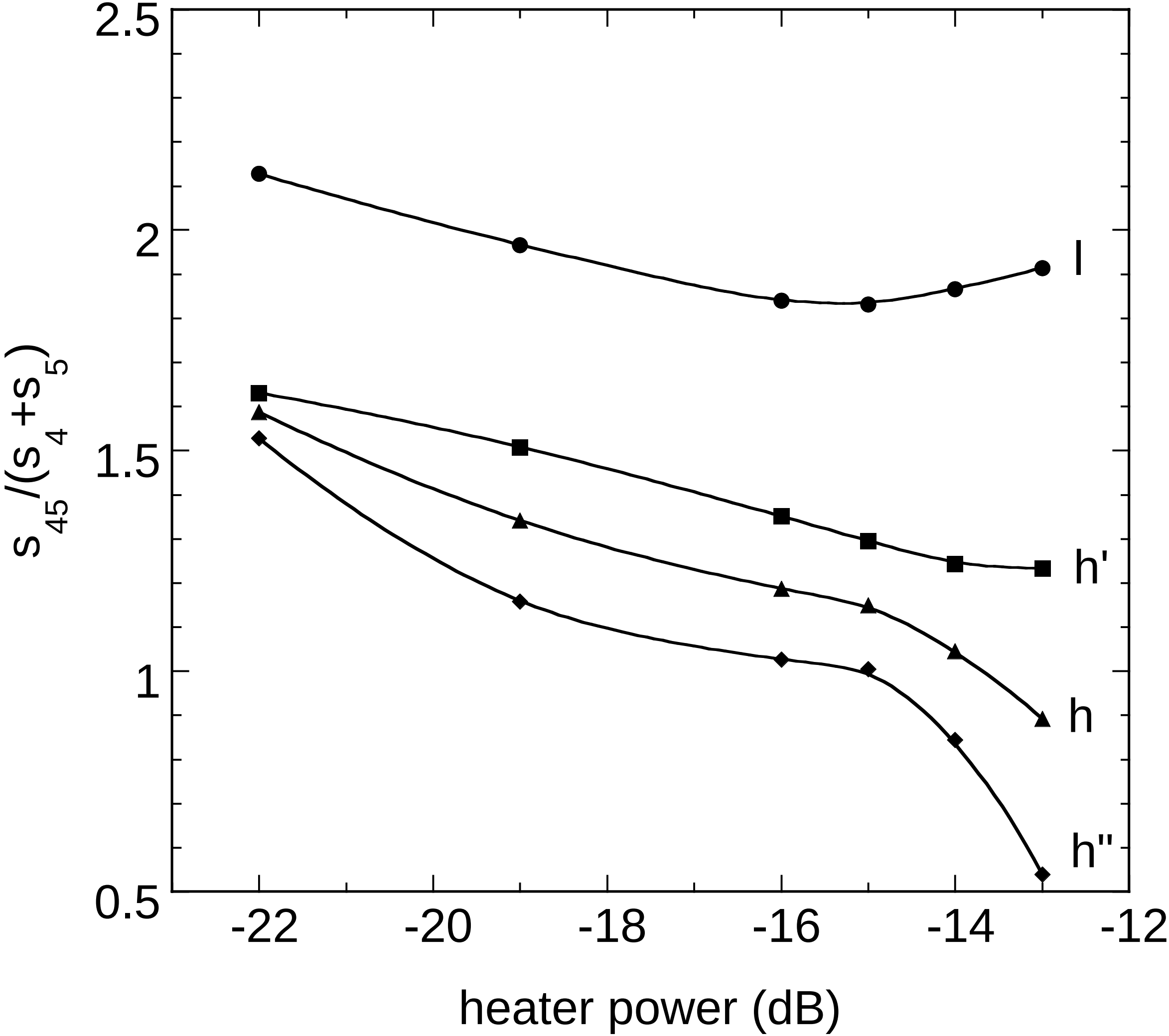

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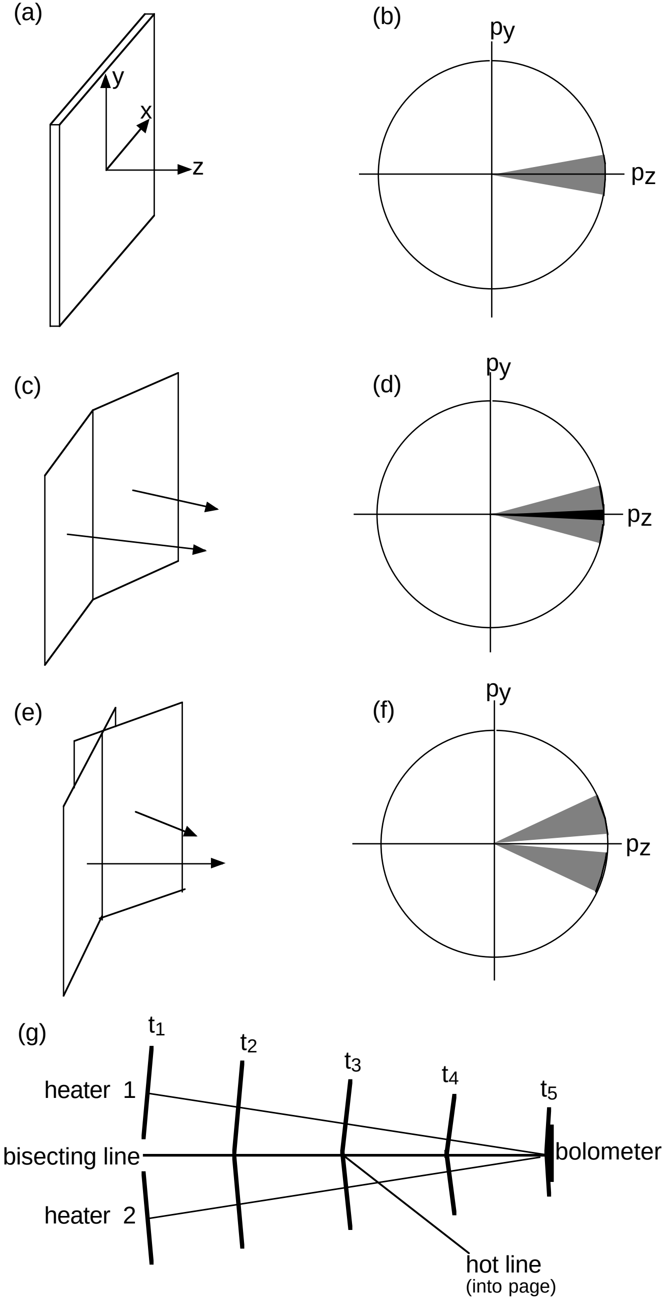
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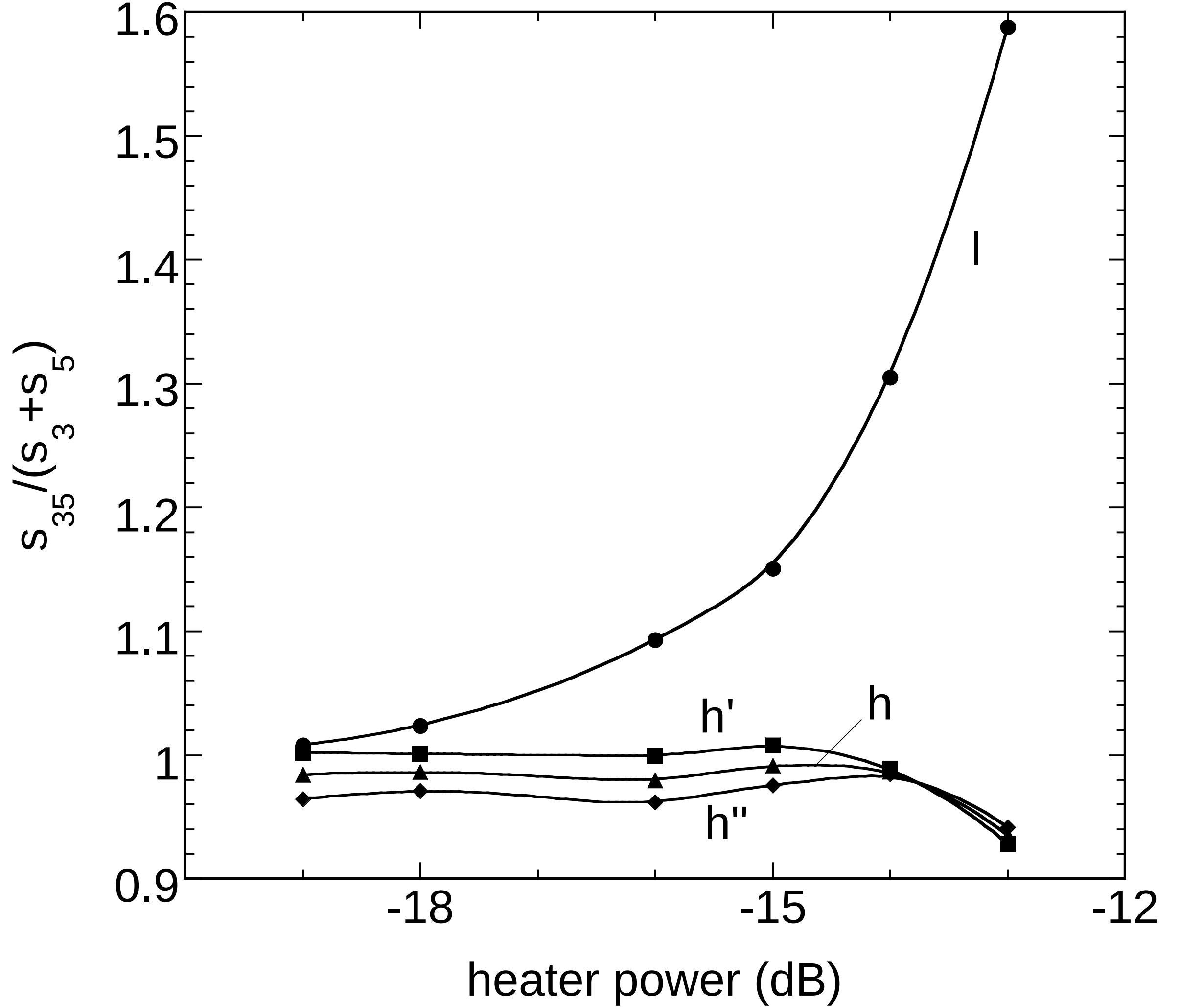

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