Pressure dependence of interacting phonons in Liquid ^{4}He

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Abstract

We have measured the pressure dependence of a single phonon sheet and the interaction of two such sheets, in liquid helium, from 0 to 21 bar and show it is dominated by three phonon processes (3pp). Pressure affects the 3pp scattering by changing the shape of the dispersion curve. The scattering varies from very strong at $P = 0$ to zero at $P = 19$ bar. The 3pp is small angle scattering at $P = 0$ and, as pressure is increased, the angles decrease further and eventually become zero. We find that the signal from a single phonon sheet increases considerably with pressure to a maximum at $P = 7.5$ bar and then decreases to a minimum at ~ 15 bar, and becomes independent of pressure at > 19 bar. We discuss this behaviour in terms of the effect of 3pp on the cone of occupied states in momentum space, the expansion of the phonon sheet and the creation of high energy phonons. The signal from the collision of two sheets, at 8.8° to each other, largely follows the pressure dependence of the energy density in the individual sheets but there is also direct evidence that the interaction between two phonon sheets is caused by 3pp scattering. In the following paper, [1], the pressure dependence of colliding sheets is considered theoretically.

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1 Introduction

Low energy phonons with energy $\epsilon/k \sim 1$ K can be created in liquid helium by a thin film heater and can form a strongly interacting and anisotropic system. They have many interesting properties such as creating another phonon system of much higher energy phonons with $\epsilon/k > 10$ K [2], forming phonon sheets [3] and creating hot lines when two sheets collide [4]. These phenomena can be understood in terms of the phonon scattering processes. The low energy phonons, l-phonons, are usually considerd to scatter by three phonon processes (3pp) but direct evidence for this is desirable. Scattering by 3pp is fast, involves small angles and allows spontaneous decay [5, 6, 7]. The high energy phonons, h-phonons at energies $\epsilon > \epsilon_c$, are created by four phonon processes (4pp). Spontaneous decay processes for h-phonons, are not allowed. The collision and interaction between two phonon sheets is also considered to involve 3pp. This paper is concerned with controlling the 3pp scattering with pressure P, so we can test these assumptions.

Phonon scattering by 3pp and the existence of the critical energy $\epsilon_c(P)$ arises from the shape of the dispersion curve $\epsilon(p, P)$. It initially bends upwards at $P = 0$ [8], so-called anomalous dispersion. If we write

$$\epsilon = cp(1 + \psi(p, P)) \tag{1}$$

then $\psi(p, P))$ is positive for $0 < p < p_c(P)$, and negative for $p_c(P) < p < p_{mxn}(P)$, where $p_c(P)$ is the momentum at $\epsilon_c(P)$ and $p_{mxn}(P)$ is the momentum of the maxon of the dispersion curve, $p_{mxn}/\hbar = 1.13$ Å^{-1} at pressure $P = 0$. When $\psi(p, P) > 1$, phonon scattering processes which change the number of phonons, are allowed as energy and momentum can be conserved. For example, when $p <\sim 0.8p_c$ (the numerical factor depends on the exact shape of $\psi(p, P)$), one phonon can decay into two phonons. Momentum is conserved by the two product phonons having a non-zero angle between their momentum vectors, as the sum of the moduli of their momenta is more than the momentum of the initial phonon.

The shape of the dispersion curve varies with pressure. The low frequency sound velocity c increases with pressure, due to the compressibility increasing faster than the density. Also $\psi(p, P)$), and hence $\epsilon_c(P)$, changes dramatically with pressure [9]. Indeed, by P =19 bar, $\psi(p, 19\ bar)) < 0$ for all p and $\epsilon_c(19\ bar) = 0$ [10, 11]. So, for pressure $P > 19$ bar there cannot be three phonon processes.

Increasing the pressure gives us a way of slowly turning off 3pp. We can investigate the variation of single phonon sheets and collisions between sheets as the 3pp interactions between low energy phonons are changed from very strong at $P = 0$, to zero at $P = 19$ bar. We shall find, as we have previously assumed [2, 12], that 3pp scattering is central to the behaviour of low energy phonons. This is seen most clearly in the collisions between two phonon sheets [4]. When 3pp scattering is impossible, there is no interaction between two phonon sheets and they just pass through each other. The results show that single sheets have a large pressure dependence and new ideas are suggested to explain them.

In this paper we describe the experiment in section 2 and give the results for single sheets and discuss them in section 3 and for colliding sheets in section 4. We draw conclusions in section 5. In the following paper [1] the pressure dependence of colliding phonon sheets are considered theoretically.

2 The Experimental Arrangement

The experimental arrangement is similar to that used to measure the angular dependence of the formation of the hot line [14] except that in the present case, the bolometer was positioned at the centre of curvature of the cylindrical lens. This means that the centre of the phonon beam from all heaters, hits the centre of the bolometer at all pressures. This is important because the lateral width of these beams are expected to change with pressure, and if the bolometer is not at the centre of curvature, then it detects phonons which are not in the centre of the beam and so would give a misleading variation with pressure.

We fabricated accurately positioned heaters by evaporating gold film heaters onto a cylindrical glass lens and then defining the individual heaters by lines scratched through the gold film, see the inset in figure 1. The lens had a radius of curvature of 12.9 mm and the line of the heaters was perpendicular to the cylindrical axis. The heaters were 1 mm$\times$1 mm and the angle between the normals of adjacent heaters was 4.4°. Current pulses of 100 ns duration were applied to a heater from a pulse generator (LeCroy 9210) creating pulse powers in the range 3 to 25 mW.

The bolometer detector, at the centre of curvature of the lens, was in the plane of the arc of heaters. The bolometer was a zinc film, 1 mm$\times$1 mm, cut into a serpentine track with resistance at room temperature $\sim 300\ \Omega$. At low temperatures this was held at $\sim 50\ \Omega$, on the superconducting transition edge, by a feedback circuit, [15]. The superconducting transition temperature was lowered to ~ 350 mK with a constant external magnetic field. The change in feedback current to the bolometer was proportional to the power absorbed. The signal was amplified by a broad band, dc to 1 MHz, amplifier (EG&G 5113) and then recorded with a Tektronix DSA 601A. Many repetitions were averaged to improve the signal to noise ratio. The responsivity of the detection system is 6.03×10^3 VW^{-1}.

The experimental cell was cooled by a dilution refrigerator to ∼50 mK. The cell was filled with isotopically pure ^{4}He [16]. The pressure is measured with a Druck pressure gauge.

3 A single phonon sheet

3.1 Results for a single sheet

For the pressure measurements we used the three most central heaters, H6, H7 and H8. They all gave similar results. Typical bolometer signals, at various pressures, are shown in Figure 1, for a heater pulse of 100 ns duration and 12.5 mW power. The signals at low pressures show two distinct contributions, the narrow l-phonon peak that arrives first and the broader h-phonon peak that arrives later. It is clear that as the pressure is increased, the signal arrives earlier due to the velocity increasing with pressure. The separation between l- and h-phonon peaks reduces with pressure due to the deviation, $\psi(p, P)$, from a linear dispersion curve, decreasing. Another effect is that the l-phonon peak height varies with pressure. This is shown in Figure 2 for four heater powers, all with 100 ns pulse length.

In figure 2 the peak l-phonon signal, S, divided by the heater power, W_h, is plotted against pressure. The integral of the l-phonon signal behaves similarly to the peak height. A number of features are apparent. We see that the l-phonon signal rises with increasing pressure to $P \sim 7.5$ bar, then it decreases reaching a minimum at ~ 15 bar . Thereafter, it rises a little and saturates within the random error on the data. This pattern is shown for all heater powers, but for the lowest heater power, 3.2 mW, the magnitude of the changes with pressure, are much smaller. At both $P = 0$ and $P = 21$ bar, the l-phonon signal is approximately proportional to the heater power for all heater powers. However the fraction of heater power that is detected decreases a little with heater power. This has been seen before at 0 bar [3]. As it occurs at 21 bar as well as 0 bar, it suggests that it is due to a greater fraction of the heater power being lost to the heater substrate at higher powers, and not due to the propagation in the liquid helium or the bolometer. Finally, we see in figure 2, that S/W_h at 21 bar is a little more than twice its value at 0 bar and we discuss this later.

As the h-phonon signal is extended in time, we consider the integral of the h-phonon signal. However there is a difficulty as the h- and l-phonons are not well separated in time, at the higher presures. So we adopt the following scheme; the signal at 18.9 bar should be only due to l-phonons and this is integrated over the period of 50 μs from the start of the signal. To obtain a value for the integrated l-phonons at lower pressures, the integral at 18.9 bar is scaled according to the l-phonon peak height at each pressure. Also at each pressure, the total signal is integrated over 50μs from the start of the l-phonon signal, and the integrated contribution from the l-phonons, as calculated above, is subtracted from the total integral. This difference we take as the integrated h-phonon signal. Clearly there is the possibility of a systematic error, however we believe the result that there is a general decrease in the h-phonons with pressure, is very reliable. In Figure 3, we show the integrated h-phonon signal as a function of pressure for different heater powers. The signal decreases with pressure for all powers which indicates that there is a strong overall decrease in the production of h-phonons as pressure is increased.

3.2 Discussion of the effects of pressure

To understand the pressure dependence of the detector signal, it is necessary to survey the various effects of pressure on the heater, phonon scattering in the helium and the bolometer. The transmission of phonons from the heater to the helium is efficient and transfers most of the heater pulse energy into the helium. A small fraction of the heater energy goes into the substrate supporting the thin film gold heater. Most of the energy goes into the liquid helium via the background channel [17, 18, 19] and, although there is no evidence on the pressure dependence of this channel, it might be expected that a slightly higher fraction of the energy goes into the liquid helium when the pressure is increased from zero, as the helium becomes better acoustically matched to the heater.

In contrast to the heater, where there is a high phonon transmission probability into the helium, the transmission of phonons from a beam into the bolometer has a low probability. Typically, the probability is around 10^{-3} for phonons with energy $\epsilon/k \sim 1$ K [20, 21]. The transmission is both by the acoustic channel and the background channel. For a rough bolometer surface the background channel is more important as the incident phonons are approximately in one direction. The transmission probability t by the acoustic channel is

$$t \sim 4z_h z_z/(z_h + z_z)^2 \tag{2}$$

where $z_i = \rho_i c_i$ and ρ_i and c_i are the density and sound velocity, and subscripts h and z refer to the helium and the zinc film of the bolometer, respectively. For zinc, the transverse sound velocity is appropriate as the density of states of transverse phonons is much higher than that for longitudinal phonons. For modest pressure changes, $0 < P < 20$ bar z_z is unchanged, but z_h increases by $\sim 60\%$. Between 0 and 7 bar the change is $\sim 27\%$. Again, there is no information on the pressure dependence of the background channel, but if we assume that it is the same as the acoustic channel, then t, and therefore the responsivity of the bolometer, only changes by $\sim 27\%$ up to 7 bar. As the measured increase in the l-phonon signal between 0 and 7 bar, is a factor between 2 and 4, depending on pulse power, see figure 2, we rule out the bolometer being responsible for this large change.

For the higher energy h-phonons going into the bolometer, the situation is more subtle. The main transmission channel for these phonons is via the background channel which is strongly dependent on the energy of the phonons. At $P = 0$ bar, it increases linearly with the phonon energy ϵ, to 5×10^{-3} for $\epsilon/\mathrm{k} = 5$ K and then remains constant [19]. As the energy of the h-phonons decreases below $\epsilon/\mathrm{k} = 5$ K, due to ϵ_c decreasing with increasing pressure, the response of the bolometer decreases for the same energy flux. Any increase in the responsivity with pressure would act in the opposite direction. At 10 bar $\epsilon_c = 5$ K, so the decrease in the h-phonons signal with pressure for $P >\sim 10$ bar, underestimates the decrease in the energy flux of the h-phonons, although, as the signal is decreasing rapidly in this range, see Figure 3, the change in responsivity makes only a small effect, it would raise the energy flux, derived from the signal, at $P > 10$ bar.

Phonon scattering in liquid helium depends strongly on pressure. The 3pp is very rapid, $\sim 10^{10}$ s^{-1}, at $P = 0$ [7] and drops to zero by 19 bar. The maximum 3pp half-energy angle decreases with pressure, from $< 11°$ at $P = 0$ [12], as the upward dispersion reduces and the scattering becomes more colinear. The 4pp scattering rate for $\epsilon > \epsilon_c$ changes because $\epsilon_c(P)$ decreases and because the dispersion curve becomes more linear around ϵ_c. There are no theoretical calculations of 4pp rates as a function of pressure, at present. It is, however, clear that the 4pp scattering rates do not vanish at any presure. This means that at some pressure, $P < 19$ bar, the 3pp rate is equal to the 4pp creation rate and then the creation rate of h-phonons is limited by the 3pp rate.

At $P = 0$, the phonons emitted by the heater into the liquid helium form a phonon sheet of strongly interacting l-phonons which occupy a small solid angle in momentum space [3]. The temperature T and the occupied solid angle $\Omega^{(l)}$ are initially determined by the heater pulse power. Subsequently, $\Omega^{(l)}$ remains constant but T decreases due to h-phonon creation and the lateral expansion of the sheet [13, 14]. The lateral expansion rate decreases, as pressure increases, due to the reduction in the 3pp scattering angles. At the limit when the scattering is colinear, the lateral expansion is zero. We suppose that the behaviour at $P = 0$, persists at $P > 0$, and continues up to a pressure where the 3pp scattering angles are too small to create a strongly interacting system of phonons. We argue below, that this pressure is ~ 7 bar.

We believe that the behaviour of the l-phonon signal with pressure, for $0 < P < 19$ bar, is determined by the change in the shape of the dispersion curve with pressure and the effect that this has on the 3pp scattering angles and rates, and the consequential effect that this has on the formation and lateral expansion of the sheet. There is a secondary effect due to the creation of h-phonons. The sensitivity of the bolometer to phonon energy has only a small effect on the l-phonon signal.

3.3 Discussion of a single sheet

We now describe the complex set of interactions which we believe is the explanation of the pressure dependence of the l-phonon signal. The most surprising feature of the results in figure 2 is the rapid decrease at $P \sim 7.5$ bar and the minimum at $P \sim 15$ bar. The explanation of the first feature is the most speculative, and perhaps, the most interesting.

We start with the injection of phonons into the helium by the heater. The heater temperature can be estimated from the measured boundary resistances [17] and approximately varies between ~ 1.7 and ~ 2.6 K for heater powers between 3.2 mW and 25 mW. The spectrum of phonons transmitted into the helium will be at lower energies than those associated with these temperatures, because the main transmission channel is the background channel where phonon energy is not conserved. Phonon energies are approximately halved on transmission, as one phonon in the heater creates two phonons in the helium [22].

The phonons are emitted into a wide angular range because the dominant background channel emits a wide angular distribution. Also, the surface of the heater film is rough on the nm scale. This angular distribution could be measured at 20 bar where the phonons travel ballistically to the detector but this has not yet been done, however we know that it must be done with short pulses as long pulses behave differently [23].

At zero pressure the emitted phonons form a sheet of strongly interacting phonons. This happens immediately after they are emitted as the 3pp scattering time of $\sim 10^{-10}$ s [7] is much shorter than the pulse length, 10^{-7} s. The angular distribution of these phonons has been measured [3] and has a mesa shape. The flat top of the angular distribution indicates a constant energy density over an area much greater than the 1mm^2 of the heater. The sides of the mesa indicate that the energy density decreases with angle. In real space, the sheet is planar in the centre, over an area equal to that of the heater, and then is gently curved, with the radius of curvature equal to the distance from the heater.

The shape of the mesa is due to two factors. The sheet expands laterally due to 3pp scattering. The theory of this expansion is similar to the expansion of a gas cloud into a vacuum [13]. There is little expansion along the direction normal to the sheet. The lateral expansion lowers the energy density in the sheet more at the edges of the sheet than at its centre, so the energy density decreases smoothly from the centre. The second factor is the creation of h -phonons within the sheet [2]. These phonons leave the sheet because their group velocity is lower than that of the sheet. This causes the energy, and hence the energy density of the sheet, to decrease.

The creation of h-phonons depends strongly on the temperature of the phonon sheet and drops to a low value at $\sim$ 0.7 K at $P = 0$ [12]. So after a distance of $\sim$ 10 mm all parts of the sheet that had a temperature higher than 0.7 K have cooled to this temperature. This region of constant temperature forms the top of the mesa [3]. The sides of the mesa are at a lower temperature than 0.7 K, because of lateral expansion.

The energy density in the sheet, $E^{(l)}$, is not only due to temperature but is also proportional to the solid angle $\Omega^{(l)}$ of the occupied cone of states in momentum space. This is formed in the initial creation of the sheet and remains at the initial value thereafter. When the sheet expands laterally, the magnitude of the solid angle of the occupied cone remains constant but the axis of the cone rotates in momentum space so that the local cone axis is always parallel to the surface normal of the phonon sheet [13].

The value of $\Omega^{(l)}$ increases with the heater power but more slowly at higher powers [14]. The effect of $\Omega^{(l)}$ can clearly be seen in the energy density of sheets created with different heater powers, after they have all cooled to the same temperature of $\sim$ 0.7 K [3]. The measured energy density is found to increase with the heater power. This is due to different values of $\Omega^{(l)}$ and not of T. The mechanism for the formation of the initial phonon sheet and $\Omega^{(l)}$ is, as yet, unknown. The formation process seems to collect phonons which have a large angular spread and form them into a narrow cone whose angle corresponds to $\Omega^{(l)}$. One possibility is that it is due to the quantum bracket, in the expression for the collision integral [7], which enhances transmission into occupied states, which have momentum predominately in the normal direction.

The behaviour described above is for $P = 0$. We believe that the pressure dependence in the range 0 –7 bar is similar to that at 0 bar. The important additional factor is that the lateral expansion diminishes with pressure. This is caused by $\Omega^{(l)}$ decreasing with pressure, at constant power, which is a result of the 3pp scattering angles decreasing with pressure. This behaviour ultimately stems from the dispersion curve changing shape as discussed above. The range of momentum with upward curvature and the deviation from linearity, $\psi(p, P)$, both decrease as pressure increases.

The velocity of the lateral expansion is given by [13]

$$v = c(\Omega^{(l)}/4\pi)^{1/2} \tag{3}$$

The sheet area at the bolometer is equal to the area of the heater plus that due to the expansion which is proportional to $\Omega^{(l)}$. With increasing pressure, $\Omega^{(l)}$ decreases and so the lateral expansion is less, hence the area is less and the energy density is proportionally higher. However, the energy density in the sheet, $E^{(l)}$ is proportional to $\Omega^{(l)}T^4$ and so for the signal to increase with pressure, the final temperature must increase because $\Omega^{(l)}$ decreases with pressure. Hence the h-phonon creation rate must decrease with pressure if the sheet temperature is not to fall to ~ 0.7 K, the value at $P = 0$. This is confirmed by figure 3 which indicates that the h-phonon creation does decrease with pressure.

We now estimate the increase in the final temperature in the sheet, when $P \sim 7.5$ bar. Figure 2 shows that the signal increases by a factor of up to ~ 4. If the sensitivity of the detector increases as T [19] then the signal varies as $E^{(l)}T \propto \Omega^{(l)}T^5$. Therefore the increase in T must be a multiplicative factor greater than $4^{1/5} = 1.3$ to counteract $\Omega^{(l)}$ decreasing with pressure.

The above picture for $0 < P < 7$ bar, that $\Omega^{(l)}$ decreases with pressure, is supported by the behaviour with heater power. In figure 4 we show the ratio of the l-phonon signals at 6 and 0 bar, and also at 3 and 0 bar, as functions of heater power, W_h. We see that the ratio extrapolates to unity at zero power then rises with power and saturates at the highest powers. We can understand this behaviour phenomenologically. We write the area of the sheet $A(P)$ at pressure P, at the bolometer distance, as

$$A(P) = A_c + m(P)f(W_h) \tag{4}$$

A_c is a constant, approximately equal to the area of the heater, $m(P)$ is a proportionality factor that depends on P and $f(W_h)$ is some function, as yet unknown but approximately linear, which is zero when $W_h = 0$ and increases with W_h. The signal $S(P)$ is proportional to the energy density which is inversely proportional to the area, hence

$$\frac{S(6\text{ bar})}{S(0\ bar)} = \frac{A(0\text{ bar})}{A(6\text{ bar})} = \frac{A_c + m(0\text{ bar})\mathrm{f(W_h)}}{A_c + m(6\text{ bar})\mathrm{f(W_h)}} \tag{5}$$

This equation has the asymptotic behaviour shown in figure 4; when $W_h \to 0$, $S(6)/S(0) \to 1$, and when $f(W_h)$ is large $S(6)/S(0) \to m(0)/m(6)$ which is a constant. We can now make the following deductions. As the increase in sheet area is given by $m(P)f(W_h)$, which is proportional to $\Omega^{(l)}$, by equation 3, we see that at a given W_h, $\Omega^{(l)}(0)/\Omega^{(l)}(6) = m(0)/m(6)$. As $S(6)/S(0) > 1$, see figure 4, then $m(0)/m(6) > 1$ and so $\Omega^{(l)}(0)/\Omega^{(l)}(6) > 1$. We see equation 5 that $\Omega^{(l)}(P)$ decreases as pressure increases. Also we see that as $W_h \to 0$, $S(0) = S(6)$ hence $A(0) = A(6)$, i.e. the area of the sheet is independent of pressure as $W_h \to 0$. Furthermore there is no expansion in this limiting case because the increase in area $m(P)f(W_h) = 0$ because $f(W_h) = 0$, hence $\Omega^{(l)} \to 0$ as $W_h \to 0$. The conclusion is that $\Omega^{(l)}$ increases with heater power and decreases with pressure in the range $0 < P < 7$ bar.

At $P > 7.5$ bar the signal rapidly decreases with pressure and we enter a new regime. We suggest that this is because the sheet starts to expand again. We speculate that the formation of $\Omega^{(l)}$, with a decreasing value as pressure increases, breaks down at $P \sim 7$ bar. This could happen if the 3pp interactions are nearly colinear. When the interactions have a very small angle, the interaction amongst the phonons at different points on the sheet, becomes weak and each part of the propagating front becomes nearly independent of its neighbour. This is a pseudo ballistic regime, with phonons propagating in approximately straight lines, but nevertheless, interacting strongly by 3pp in colinear scattering.

As a consequence, the phonon sheet broadens as pressure increases and it tends to the angular distribution of the phonons emitted by the heater. This would be attained completely at 19 bar. So we propose that from 7 bar to 15 bar, the phonon sheet expands continuously with a consequent drop in energy density. The concepts of $\Omega^{(l)}$ and T become increasingly inapplicable in this range of pressures; the phonon spectrum is now determined by the injected spectrum and the subsequent 3pp interactions, so as the 3pp rate becomes small compared to the inverse of the propagation time, the spectrum approaches the injected one.

The minimum at 15 bar suggests that the spectrum of phonon energies has shifted to energies below that injected. This can be caused by 3pp decay processes. The large angular range suggested above, means that momentum space is sparsely occupied and this prevents up-scattering balancing decays, within the propagation distance. The lower energy spectrum gives a lower signal from the bolometer due to the lower responsivity at lower phonon energy.

For $15 < P < 19$ bar, the 3pp scattering becomes so weak that the spectrum shifts to higher energies and becomes the emitted spectrum by 19 bar, where 3pp scattering ceases. At $P > 19$ bar, the signal does not change as the phonon propagation is fully ballistic, with phonons propagating from heater to bolometer without scattering.

The larger signal at 20 bar compared with that at 0 bar could be due to: (i) the creation of h-phonons at 0 bar but not at 20 bar, (ii) different angular distributions at the two pressures and (iii) different phonon spectra giving different responses in the bolometer. It is not clear at the moment which is the dominant factor and a new experiment will be needed to resolve this.

4 Colliding phonon sheets

The importance of 3pp scattering can be established by colliding phonon sheets. Two phonon sheets can be created by simultaneously pulsing two heaters. We used H6 and H8 which have an angular separation of 8.8°. When two phonon sheets collide, a hot line is created and the l- and h-phonon signal increases [4]. The l-phonon signal is maximised at each pressure by slightly delaying one of the heater pulses with respect to the other. The maximum delay was < 1 μs. This ensures that the hot line is fully incident on the bolometer. The fact that the response of the hot line to pressure is similar to that of a single sheet, gives confidence that the single sheets were also symmetrically disposed about the bolometer.

The pressure dependence of these signals is shown in figure 5. The l-phonon signal shows a similar behaviour to that of a single sheet. With increasing pressure, the signal rises to a peak at ~ 7.5 bar and then falls, to below its value at $P = 0$, at ~ 15 bar. (This is a greater fall than for a single sheet). At higher pressures, it rises a little to a constant value which is just equal to the sum of the two signals from the separate sheets at this pressure.

This behaviour of the colliding sheets is mainly determined by the energy density in these phonon sheets, $E^{(l)}$, when they are near the bolometer. There is also the influence of the 3pp scattering rate becoming so low that the sheets do not interact even though the energy density in the sheets is not zero. The size of the l-phonon signal from the hot line, is determined by the rate of energy being fed into the hot line by the sheets [14]. This rate is proportional to $E^{(l)}$ sinα where α is the angle between the sheets. This applies for $\alpha < 13°$ at 0 bar [14], so we kept α constant at 8.8° when the pressure was varied. The angles and rates for 3pp scattering must be large enough for strong interactions between the phonons in the two sheets. For a strong interaction between the two sheets, the two corresponding occupied cones in momentum space, must not be separated by more than the 3pp angle for decay into two equal phonons.

For $0 < P < 7$ bar, the l-phonon signal from the hot line follows $E^{(l)}$. This behaviour can be seen when we compare heater powers 3.2 mW and 6.3 mW. The weaker response to pressure at 3.2 mW can be seen in both the single sheet and the colliding sheets. For $7 < P < 15$ bar, the hot line decreases with $E^{(l)}$, but the fall off in 3pp scattering rate also contributes to the rapid decrease in the hot line signal between 10 and 15 bar: it can be seen in figures 2 and 5, that the hot line signal falls faster than the signal from a single sheet. The l-phonon signal from the hot line alone, is essentially zero by 19 bar even though E^l is still large. For $P >\sim 19$ bar, the two sheets do not interact but just pass through each other so that the signal from two simultaneous pulses is equal to the sum of the signals from separate pulses, as shown in figure 5 and more clearly in figure 6.

In figure 6 we show the ratio of the l-phonons in the hotline to the sum of the l-phonons in the separate sheets, as a function of pressure for the four heater powers. We see that the ratio is constant at ~ 3.8 for $0 < P < 7$ bar and then it decreases to unity at ~ 19 bar where there is no interaction between the sheets. The behaviour is independent of heater power and does not show the strong variation with pressure that is shown by both the single sheet and the hot line. The behaviour in figure 6 confirms that the 3pp scattering causes the interaction between the sheets.

5 Conclusions

We have measured the signal from single phonon sheets and from two sheets colliding at 8.8° to each other, as a function of pressure, for four different powers, in order to see the effect of turning off the three phonon processes. We find that pressure has a large effect on single sheets, with the signal rising between 2 and 4 times to a peak at ~ 7.5 bar and then falling to about twice the value at 0 bar, by 19 bar. For colliding sheets there is a similar increase to 7.5 bar and then the signal falls to the sum of the signals from separate sheets by 19 bar, showing that the sheets have stopped interacting.

These pressure measurements have confirmed our previous ideas on the creation and behaviour of phonon sheets and collisions between sheets, and it has extended our understanding and raised new questions. The most basic idea that 3pp scattering between l-phonons is central to their behaviour, has been demonstrated beyond doubt. This is most clearly seen in figure 6 where we see that two phonon sheets do not collide, but pass through each other at high pressures when the 3pp is not allowed.

Our picture of the phonon sheet is quite involved. We envisage that the phonons, from a short heater pulse, are emitted into the helium over a wide, but as yet unknown, angular range and at least some of these phonons are gathered up into a narrow angular range to form an occupied cone in momentum space, with solid angle $\Omega^{(l)}$. The boundaries of this cone are not sharply defined, as discussed in reference [1], although it has been practical to use a well defined cone in previous analyses. The value of $\Omega^{(l)}$ is determined in the initial stages of the formation of the sheet and its value increases with heater power but the formation process is not yet understood. As the sheet propagates from the heater, it expands and the increase in area is proportional to $\Omega^{(l)}$.

The present results indicate that $\Omega^{(l)}$ decreases with pressure, at a given power, between $0 < P < 7$ bar. This conclusion is necessary to explain the increase in signal along the normal to the heater and is based on the following line of reasoning. From theory [13] the velocity of lateral expansion is proportional to $(\Omega^{(l)})^{1/2}$. If $\Omega^{(l)}$ decreases, there is less expansion and so the energy density of the sheet increases. Also if there is less energy loss due to h-phonon creation, as shown in figure 3, the final temperature increases, and hence the final energy density increases which gives a higher signal. Physically this is reasonable. As pressure increases, the 3pp angles decrease, because the dispersion is less, and the cone of occupied states in momentum space has a cone angle which must be greater or equal to the 3pp half-energy angle [1].

This picture of $\Omega^{(l)}$ decreasing with pressure, is supported by the ratio S(6 bar)/S(0 bar) increasing with power, as shown in figure 4. As power increases, $\Omega^{(l)}$ increases but the rate of increase is lower for higher pressures. So at a given heater power, the sheet area is smaller at higher pressures and so the energy density and signal are higher. As the heater power goes to zero , the ratio tends to unity which implies that there is no expansion at either $P = 0$ or at $P = 6$ bar when $W_h \to 0$, and all the sheets have the same area, independent of pressure for $0 < P < 7$ bar, in this limit.

When $P > 7$ bar we enter a new regime. The signal decreases rapidly with pressure and we have speculated that this is due to the sheet expanding again. We suggest that this happens because the process, that forms $\Omega^{(l)}$ and T, breaks down when θ_{3pp} becomes too small and the phonons form into narrow bundles of interacting phonons that are pseudo ballistic. Furthermore the angular distribution widens with pressure and reaches the angular distributution emitted by the heater, at 19 bar where 3pp cease.

The results in figure 3 show that the high energy phonon creation decreases with pressure. There is some uncertainty in the details of this graph because of the difficulty of separating the l- and h-phonons at higher pressures, see figure 1. This uncertainty could be resolved in the future, by using a detector which is only sensitive to the h-phonons.

A direct test of 3pp scattering comes from the interaction of two sheets at a small angle to each other. The l-phonon signal, from the hot line that is formed, depends on both the energy density of the sheets when they are near the bolometer, and the strength of the 3pp scattering. The overall similarity of the graphs for the single sheets and for the colliding sheets, see figures 2 and 5, shows the strong dependence of the hot line l-phonon signal on energy density. This can be best seen in the ratio of the l-phonon signals from the colliding sheets to the sum of the separate sheets, as a function of pressure, see figure 6. This ratio eliminates the peak at ~ 7 bar seen in these quantities individually, see figures 2 and 5, showing that the structure at this pressure, is similar in them both.

The decrease in the pressure range $7 < P < 15$ bar is much more for the hot line than for the single sheet, as can be seen in figures 2 and 5 and in the ratio shown in figure 6. This is direct evidence of the 3pp becoming weak at high pressure and stopping the two sheets interacting. At $P > 19$ bar the the signal fom the two sheets is just equal to the sum of the two sheets separately, which shows that no hot line is formed.

The work in this paper highlights the need to understand the formation of $\Omega^{(l)}$ in the initial stages of the sheet, and investigate the new regime that happens at $P > 7$ bar.

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Figure Captions.

Figure 1. Phonon signals as functions of time, for a single phonon sheet, are shown at different pressures. The traces have been offset vertically, but otherwise have the same scale. The inset top-left shows the schematic arrangement of the heaters H_i on the glass lens, and the bolometer B. The inset top-right shows, schematically, the phonon sheets and the hot line.

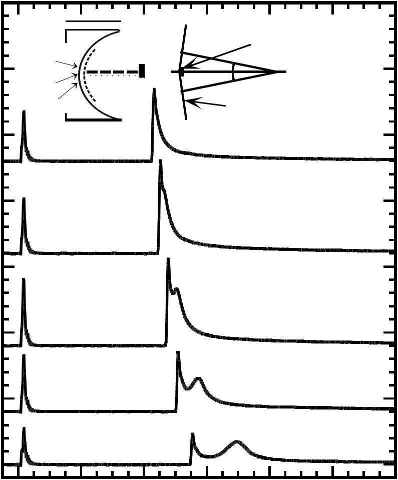
Figure 2. The peak height of the low-energy phonon signal divided by the heater power, for a single phonon sheet, is shown as a function of pressure, for different heater powers. The heater pulse length is 100ns.

Figure 3. The integrated high-energy phonon signal, for a single phonon sheet, is shown as a function of pressure for different heater powers. The heater pulse length is 100ns.

Figure 4. The ratio of the low-energy phonon signals at 6 and 0 bar, and the ratio for 3 and 0 bar,, for a single phonon sheet, are shown as functions of heater power. The heater pulse length is 100ns. Note that the curves extrapolate to 1 at zero power and saturate at higher powers.

Figure 5. The peak height of the low-energy phonon signal divided by the heater power, for two simultaneous phonon sheets, is shown as a function of pressure, for different heater powers. The heater pulse length is 100ns. The arrows indicate the sum of the signals from two separate sheets at 21 bar. As these are the same as the signal from the corresponding two simultaneous phonon sheets, at 21 bar, we see that the two sheets do not interact at this pressure.

Figure 6. The ratio of the peak height of the low-energy phonon signal, for two simultaneous phonon sheets, S_{68}, to the sum of peak heights for separate phonon sheets, $S_6 + S_8$, is shown as a function of pressure, for different heater powers to heaters H_6 and H_8. The heater pulse length is 100ns. Note the ratio goes to unity at high pressures and the ratios at different powers, behave similarly.



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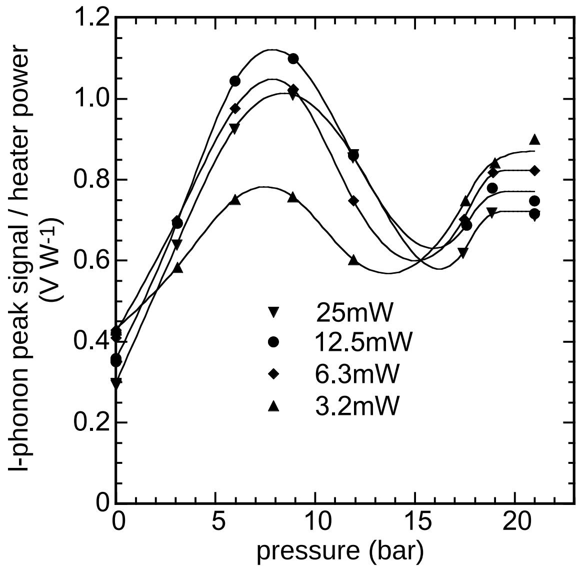
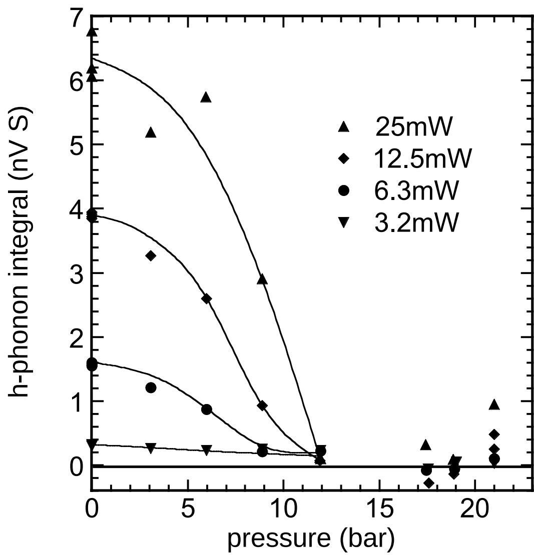
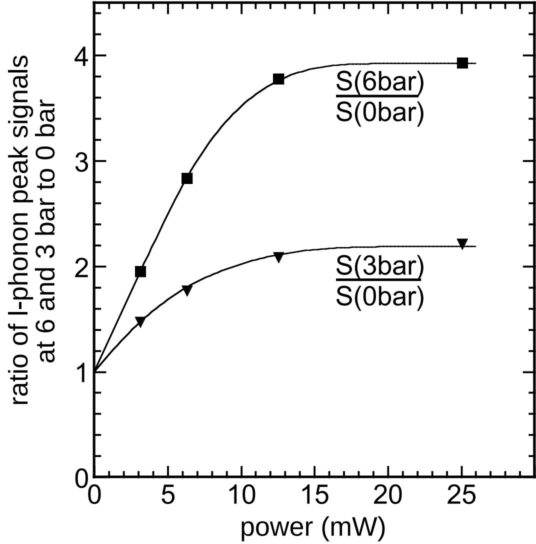
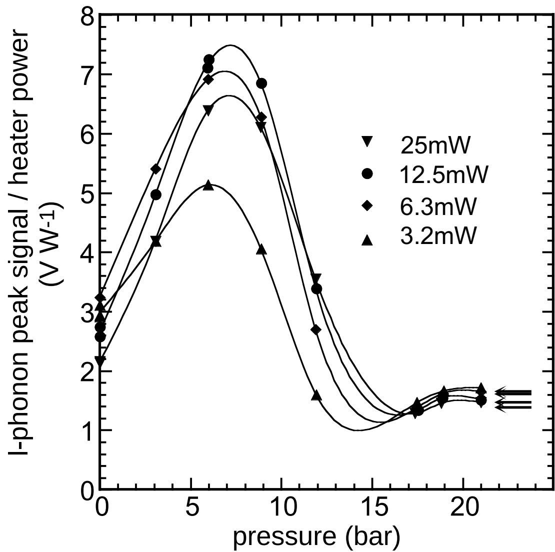


1.2
1.0
0.8
0.6
0.4
0.2
0
0
5
10
15
20
l-phonon peak signal / heater power
(V W-1)
pressure (bar)
25mW
12.5mW
6.3mW
3.2mW

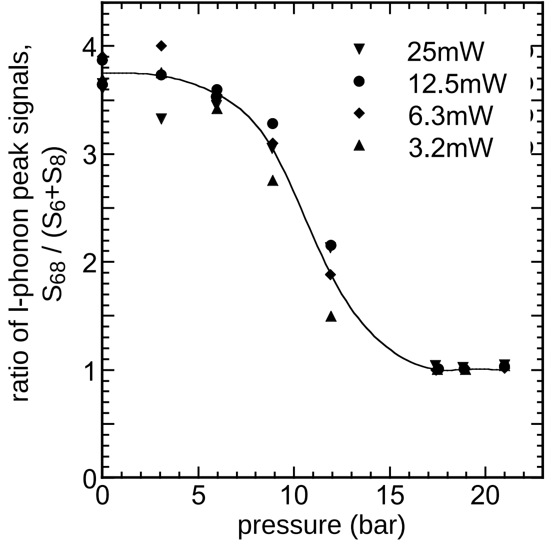

25mW
12.5mW
6.3mW
3.2mW
h-phonon integral (nV S)
pressure (bar)
0
1
2
3
4
5
6
7
0
5
10
15
20



ratio of l-phonon peak signals
at 6 and 3 bar to 0 bar
power (mW)
S(6bar)
S(0bar)
S(3bar)
S(0bar)
0
1
2
3
4
5
10
15
20
25



l-phonon peak signal / heater power
(V W-1)
pressure (bar)
25mW
12.5mW
6.3mW
3.2mW
0
1
2
3
4
5
6
7
8
0
5
10
15
20



ratio of I-phonon peak signals,
S_{68} / (S_6+S_8)
pressure (bar)
25mW
12.5mW
6.3mW
3.2mW
0
1
2
3
4
0
5
10
15
20